Performance Analysis of Free Space Optical Links Encoded Using Luby Transform codes

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Abstract— **Free Space Optical (FSO) communication is an emerging transmission technique to transmit high data rates without using cables. This technology is expected to revolutionize the present communication system architectures both in the terrestrial and the in -space architecture. Atmospheric effects can significantly degrade the performance of FSO systems. This reduces the SNR and leads to impaired performance. FSO channels can be modeled using Gamma-Gamma, Weibull, Log-Normal, K distribution functions. Error control codes can help to mitigate atmospheric turbulence induced signal fading in free space optical communication links. Luby Transform codes belong to a class of error control codes called Fountain codes and are meant for erasure channels. In this paper, we propose encoding FSO links with Luby Transform (LT) codes for error channels. Decoding is done using belief propagation with Log Likelihood Ratio and results are obtained for different modulation schemes under different channel distributions.**

Keywords- Free space optics, Luby Transform codes, scintillation, Distribution functions , Message passing, belief propagation,Log Likelihood Ratio

I. INTRODUCTION

Free space optical communications is a cost effective and high bandwidth access technique. By using a high carrier frequency in the optical range, digital communication of a very high order is possible. Also, FSO links are difficult to intercept, immune to interference or jamming from external sources .The main drawback of the FSO link is the atmospheric effect on the propagating optical beam. The atmosphere is composed of gas molecules, water vapor, pollutants and dust. Since the wavelength of the optical carrier is comparable to these molecule and particle sizes, the carrier is subjected to various effects that are not normally seen in Radio Frequency (RF) systems. One such effect is scintillation caused by atmospheric turbulence and refers to random fluctuations in the irradiance of the received optical beam [1].

Fountain codes are sparse graph codes for channels with erasures. Using digital fountains, packets can be encoded from the source data and sent across the channel continuously until the receiver has collected enough to decode the file. The receiver will need more encoded packets than they would have required without digital fountains, but they can decode the file from most random combinations of the packets. This means that lost packets do not matter as any packets which are received can be used for decoding.

The main idea of a fountain code is that the source file is encoded by an encoder which produces an endless stream of encoded packets. The source file is split to K source segments, each of length 'l'; each encoded packet will contain data of length 'l' as well. A receiver can receive these encoded packets until they have slightly more than K source segments and then they can decode the file [2]. Fountain codes are a new class of codes with finite dimension and infinite block length. An appropriately designed fountain code eliminates the need for information about the quality of the channel to a particular receiver. Universal fountain codes provide an excellent solution to the transmission problem over an unknown channel. Luby Transform (LT) codes are a class of Fountain codes.

The error performance of FSO systems is investigated assuming a fading process governed by different distribution functions such as Log-Normal, Gamma-Gamma, K and Weibull distribution and the channel distorted by additive Gaussian noise [3],[4],[5],[6],[7],[8].Error control Codes can help to mitigate turbulence induced signal fading in free space optical systems through atmospheric turbulence[9],[10].We investigate and compare the performance of FSO systems encoded with LT codes. LT codes are meant for erasure channels and give a good performance with an overhead of about 10%. We propose a novel decoding method for these codes using LLRs.

II. LUBY TRANSFORM CODES

Luby Transform codes, the first fountain codes have been investigated for their performance on Erasure Channels. The LT codes generate a limitless stream of encoded symbols which are independent. The LT decoder can recover the source data from arbitrarily collected encoded symbols with small decoding overhead. The process of LT encoding is dependent on the degree of distribution which decides the number of input symbols as the neighbours of an encoded symbol and then the encoded symbol is output as the XOR of its neighbours. At the receiver, the neighboring information of each encoded symbol is utilized to construct a Tanner Graph. The decoder uses the principle of belief propagation. The value of every degree-1 encoded symbol is propagated in each iteration to its neighbours, until all source symbols are recovered .If there is no degree-1 encoded symbol before the end of recovery, the decoding fails and it becomes necessary to collect more symbols to decode. The decoder is assumed to have knowledge of the degree and neighboring indices of each symbol [2].

The encoder needs a (k x m) generator matrix G' to create encoded symbols, where k is the code dimension and m is unlimited .The encoded symbols are produced on the fly and the Hamming weights of the columns of the generator matrix are determined by the Robust Soliton distribution proposed by Luby[11],[12]. The encoded symbols are transmitted once

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requested, but some of them are erased. At the receiver, with the n received symbols, the decoder builds a (k x n) generator matrix G and G is made by deleting those columns of G' corresponding to the erased symbols. The Robust Soliton distribution guarantees that with small overhead all source symbols are covered by encoded symbols and there is at least one degree-1 encoded symbol in each decoding iteration [13].

For all possible degrees'd' the degree distribution ,ρ(d) in the encoding algorithm is the probability that an encoded packet has degree'd'. The proper design of 'ρ' ensures that the number of packets is small.

The positive function for Robust Soliton distribution is given by

$$\tau(i) = \begin{cases} R/ik & for\ \ i = 1,...,k/R-1 \\ R\ln(r/\delta)/k & for\, i = k/R \\ 0 & for\ \ i = k/R+1,....,k \end{cases} \tag{1}$$

Where

$R = c.\ln(k/\delta)\sqrt{k}\quad for\ some\ suitable\ cons\tan t\ c > 0$

which decides the performance of the algorithm and δ is the allowable failure probability of the decoder to recover the data for a given number 'k' of encoding symbols.

The Robust Soliton Distribution is given by

$$\mu(d) = \frac{\rho(d)+\tau(d)}{Z}\ and\ Z = \sum_d \rho(d)+\tau(d) \tag{2}$$

For each value of k number of source packets, there is a value of 'c' which allows the source to be recovered using k' encoded packets, with probability (1- δ).This ensures that the decoding will complete with probability (1- δ) if k'=kZ packets are received [11].

III. Channel Model for FSO in the Presence of Scintillation

The main impediment in the free space optical channel is the scintillation. Atmospheric turbulence results in many effects causing fluctuation in the received optical power. The important effects of atmospheric turbulence on the laser beam are phase front distortion, beam wander and redistribution of the intensity within the beam. The temporary redistribution of the intensity known as scintillation results from the chaotic flow changes of air and from thermal gradients within the optical path caused by the variation in air temperature and density. Particular parts of the laser beam travel on slightly different paths and combine. The recombination is destructive or constructive at any particular moment and results in spatial distribution of signal and consequently in lowering the received power [14]. The biggest disadvantage of FSO systems is the dependence of propagation on the local weather conditions.

The intensity of the optical wave propagating through turbulent atmosphere is a random variable. The normalized variance of optical wave intensity is referred to as the scintillation index. It indicates the strength of irradiance fluctuations. For weak fluctuations, it is proportional and for strong fluctuations, it is inversely proportional to the Rytov variance for a plane wave.

The refractive index structure is dependent on the temperature, wind strength, altitude, humidity, atmospheric pressure, etc. The scintillation levels are usually divided into three regimes in dependence on the Rytov variance; a weak fluctuations regime, a moderate fluctuations regime and a strong fluctuations regime [14], [15].

It is assumed that the channel fade remains a constant during a block. Several statistical models have been proposed for scintillation. Because of its simplicity, Log-Normal distribution is the most widely used, though its applicability is restricted to weak turbulence conditions. Negative exponential model fits the strong turbulence conditions. Gamma-Gamma model describes both strong and weak turbulence regimes. This model is based on the scintillation and modified Rytov theories and considers irradiance fluctuations as the product of small scale and large scale fluctuations, where both can be statistically defined via the Gamma distribution.

The Gamma-Gamma channel model is a two parameter distribution which depends directly on the atmospheric parameters and has the ability to describe turbulence induced fading under every degree of turbulence severity. It is based on a doubly stochastic theory of scintillation and assumes that small scale irradiance fluctuations are modulated by large scale irradiance fluctuations of the propagating wave, both governed by independent Gamma distributions [4].

The Gamma -Gamma probability density function (pdf) can be directly related to atmospheric conditions and has the ability to describe turbulence induced fading under every degree of turbulence severity [16], [17]. For heavy turbulence, it is approximated using the exponential distribution, whereas in less turbulence it is suitably approximated by a Log-Normal distribution [18].

A. Gamma- Gamma Distribution

For a wide range of turbulence conditions, the fading gain in FSO systems can be modeled by a Gamma-Gamma distribution function

$$f(I) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta I}),\ I > 0 \tag{3}$$

Where $K_a(\bullet)$ is the modified Bessel function of the second kind of order 'a'. Here α > 0 and β > 0 are parameters linked to the scintillation index. If spherical wave propagation is assumed, α and β can be directly linked to physical parameters.

$$\alpha = \left[\exp\left(\frac{0.49\chi^2}{(1+0.18d^2+0.56\chi^{12/5})^{7/6}}\right)-1\right]^{-1}$$

$$\beta = \left[\exp\left(\frac{0.51\chi^2(1+0.69\chi^{12/5})^{-5/6}}{(1+0.9d^2+0.62d^2\chi^{12/5})^{5/6}}\right)-1\right]^{-1} \tag{4}$$

$$where\ \chi^2 = 0.5C_n^2 k^{7/6} L^{11/6}\ \ ,d = (kD^2/4L)^{1/2}$$

and k=2π/λ. Here λ, D, C_n^2, and L are the wavelength in meters, the diameter of the receiver's aperture in meters, the index of refraction structure parameter, and the link distance in meters respectively.

In our paper we have used α and β obtained from the physical parameters using measurement data as suggested in [4], [10].

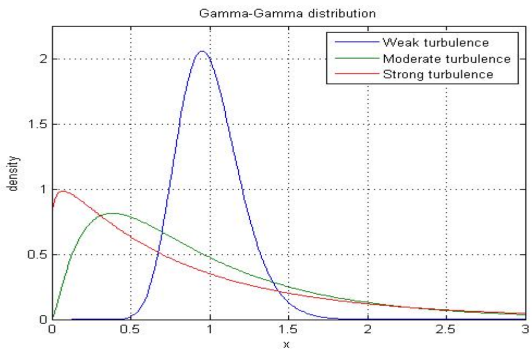

Figure 1. Gamma-Gamma Distribution

Fig 1 shows that the Gamma-Gamma distribution is symmetric for weak turbulence. But for moderate turbulence, the skewness varies. This shows that the scintillation effect tends to increase with increase in turbulence.

B. Weibull Distribution

In [16],[17], Nestor D Chatzidiamantis et al propose the Double Weibull model for describing moderate and strong irradiance fluctuations. Based on scintillation and modified Rytov's theories, composite fluctuations are considered as the product of small scale and large scale fading, both statistically described by the Weibull distribution.

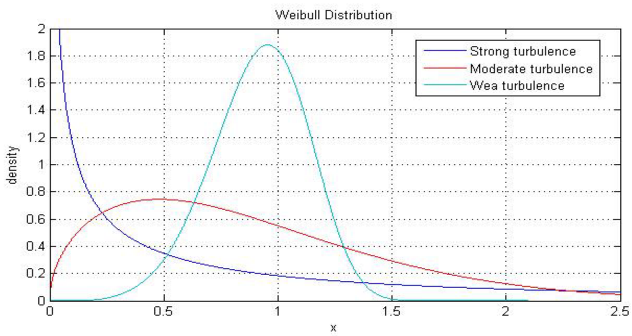

Figure 2. A plot of Weibull distribution pdf for different turbulence

The use of Double Weibull pdf as a moderate to strong turbulence pdf model is justified in [17]. It is more accurate than the widely used Gamma-Gamma pdf, but a trade off is to be made between accuracy and calculation complexity.

A Weibull distributed random variable x had its pdf given by

$$f_x(x) = \frac{\beta x^{\beta-1}}{\Omega} \exp\left(-\frac{x^\beta}{\Omega}\right) \qquad (5)$$

Where β>0 is the distribution parameter related to the severity of the irradiance fluctuations and Ω>0, is related to their average power [16]. The Double Weibull channel model is derived through the product of two Weibull random variables. The parameters of this model are linked directly to the atmospheric parameters and are derived as the product of small scale and large scale fluctuations.

IV. LT ENCODING PROCESS

The encoding operation needs the description of a graph connecting the encoded packets to the source packets. This graph, called a Tanner Graph is constructed by using the parity check matrix. Tanner graphs are bipartite graphs. This means that the nodes of the graph are separated into two distinctive sets and edges connect only the nodes of two different types. The two types of nodes in a Tanner graph are called variable nodes (v_ nodes) and check nodes (c_nodes)[19].

$$H = \begin{bmatrix} 01011001 \\ 11100100 \\ 00100111 \\ 10011010 \end{bmatrix}$$

Figure 3. H matrix to show Tanner graph

The Fig 4. shows the Tanner graph for the Matrix shown in Fig 3 .Check node f is connected to variable node c, if the corresponding element of the H matrix is a 1

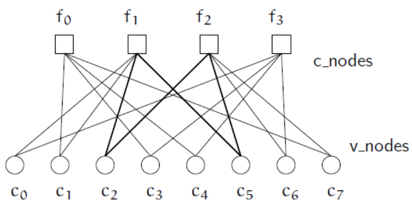

Figure 4. Tanner graph to represent the H matrix shown in Fig 3

$$C^T = \begin{bmatrix} c1 \\ c2 \\ c3 \\ c4 \end{bmatrix} = Hs^T = \begin{bmatrix} 100 \\ 111 \\ 011 \\ 101 \end{bmatrix} \begin{bmatrix} s1 \\ s2 \\ s3 \end{bmatrix}$$

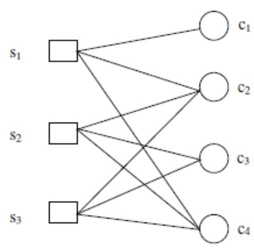

Figure 5. Generation graph for LT codes.

Each codeword symbol c_n is generated from the message bits sequence s_1, s_2, s_3…..s_k as follows [20], [21]:

1. Randomly choose a degree d_n from a degree distribution

2. Uniform randomly choose d_n message bits, bitwise sum, modulo 2 these bits and set the result as the value of c_n.

The encoding process defines a bipartite-check graph connecting the codeword symbols with original message as in Figs 3 and 4.

Since the mean degree d is significantly smaller than the message length k, this graph is sparse. The resulting code is an irregular low-density-generator-matrix code, which is similar to the Low Density Parity Check (LDPC) Codes and can be written in terms of vectors and matrices, i.e. $C^T = Hs^T$ where H is the generator-matrix corresponding to the bipartite check graph, s and c is the original message bits sequence and codeword symbols sequence respectively[20],[21].

V. DEMODULATION USING LOG LIKELIHOOD RATIOS

LT codes are normally meant for erasure channels. But we have tried to use them for error channel applications with sufficiently good results to indicate the improved BER performance. Log likelihood ratios are used for estimation since the channel is modeled using different distribution functions to indicate fading along with AWGN. The demodulator provides at its output Logarithmic Likelihood Ratios (LLR) on the transmitted bits instead of hard values of 0 or 1[10]. Soft decoding is used instead of hard decoding since it provides a better performance instead of hard decision decoding. The LLRs at the output of the decoder are fed to the channel decoder which in turn provides LLRs on the information data bits at the output.

The noise is assumed to have a Gaussian distribution and the conditional pdf of the received signal is

$$P(r \mid s) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(\frac{-(r-hs)^2}{2\sigma_n^2} \right) \tag{6}$$

Where r is the received signal and h denotes the channel turbulence, σ_n^2 is the noise variance.

The transmitted bits are 0 or 1 and for On Off Keying (OOK) modulation, we make two hypotheses on the corresponding transmitted symbol. In the case of BPSK signal, we introduce a DC shift since in optical systems; the photo detector cannot accept negative values at its input.

The decision made is based on the Maximum A posteriori (MAP) criterion.

The MAP symbol detector provides the detected symbol $\hat{s}$

$$\hat{s} = \arg\max_{s} P(r \mid s)\, P(s) \tag{7}$$

For equiprobable bits we have

$$\hat{s} = \arg\max_{s} P(r \mid s) \tag{8}$$

To obtain $\hat{s}$, we calculate the likelihood ratio (LR) as follows

$$LR = \frac{P(r \mid s_1)}{P(r \mid s_0)} = \exp\left(\frac{-(r-hs_1)^2 + (r-hs_0)^2}{2\sigma_n^2} \right) = \exp\left(\frac{2hr - h^2}{2\sigma_n^2} \right) \tag{9}$$

If LR>1 , we make the decision $\hat{s}$ =s1 and $\hat{s}$ =s0 otherwise, where s1 and s0 indicate the transmission of bit 1 or bit 0

LLR is used for decision making

$$LLR = \frac{2hr - h^2}{2\sigma_n^2} \tag{10}$$

If LLR>0, we set $\hat{s}$ =s1 and $\hat{s}$ =s0 otherwise, where s1 and s0 indicate the transmission of bit 1 or bit 0

VI. DECODING USING BELIEF PROPAGATION

In MacKay [20], [21] decoding for erasure channels is implemented using message passing algorithm. Our simulations use this algorithm but for error channels. Belief propagation involves passing of messages which are estimated probabilities. Messages are updated until convergence. Encoded packets are represented as t_n and source packets as s_n. The decoding process can be described as follows.

(1) Find a node t_n that is connected to only one source packet s_k .If there is no such node, the decoding algorithm halts at this point and fails to recover all the source packets.

- Set $s_k = t_n$
- Add s_k to all nodes t_n' that are connected to s_k
- Remove all edges that connected to the source packet s_k

(2) Repeat (1) until all {s_k} are determined.

Many packets must have low degree, so that the decoding process can get started and keep going and so that the total number of addition operations involved in the encoding and decoding is kept small.

VII. SIMULATIONS AND DISCUSSION

To evaluate the system error performance, in turbulence regimes in our simulations, the probability density function (pdf) of the received optical signal after traveling through the channel is modeled using different distribution functions. All simulations were realized in the MATLAB environment for typical FSO parameters. Since optical detectors need positive voltages, all signals represented as negative must be shifted with suitable dc bias to ensure positive values.

We have evaluated the performance of the optical wireless system encoded with Luby transform codes under different turbulence conditions with different channel models. We consider intensity modulation with direct detection for the simulations.Fig.1 shows the plot of the pdf of Gamma –

Gamma and Fig 2 that of Weibull distribution for various turbulence parameters.

The paper takes into account two simultaneously changing parameters affecting the probability of fade. The first is the received optical power which decreases with increasing link distance and the second is the scintillation index which with increasing link distance grows rapidly in the weak fluctuation regime and then decreases slowly in the saturation regime. The results presented give show the reliability of LT encoded free space communication ensured by a particular FSO deployed with different modulation schemes. The decoding is repeated for five different iterations to verify the consistency in the decoded results for LT codes. The implementations are for 50,000 and 90,000 bits of data packetized as 512 or 1024 packets of 11 bytes and repeated for 5 iterations. The LT generator matrices are generated on the fly depending on the data size.

The BER performance is assessed for different modulation schemes with Luby transform encoding for error channels. Encoding was for different data sizes and decoding was through belief propagation with LLR. We have studied the performance of LT coded FSO links. Fig 6 – Fig 11 show a plot of the BER performance for BPSK, QPSK and 16-QAM schemes encoded with LT codes, with different channel modeling distributions and the comparison shows that 16-QAM gives the best performance. It was also observed that the belief propagation method of decoding using LLR failed at some SNR values as shown in Fig 12. The performance can be improved by proper choice of channel models depending on the atmospheric parameters. We propose to classify channel models through Neural Networks using Radial Basis functions to improve performance.

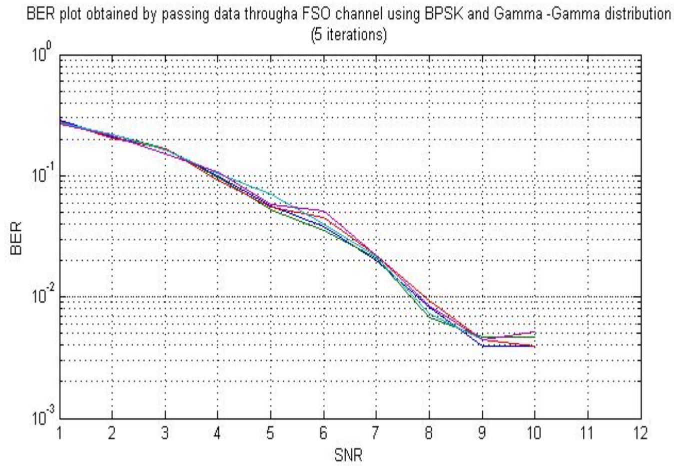

Figure 6. BER plot of BPSK modulated data through channel modeled using Gamma-Gamma distribution

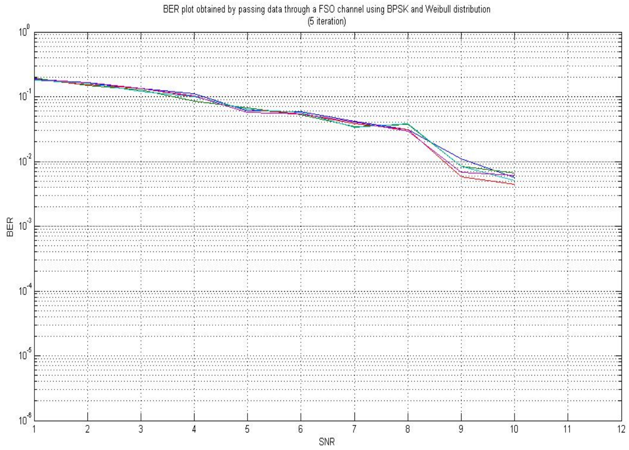

Figure 7. BER plot of BPSK modulated data through channel modeled using Weibull distribution

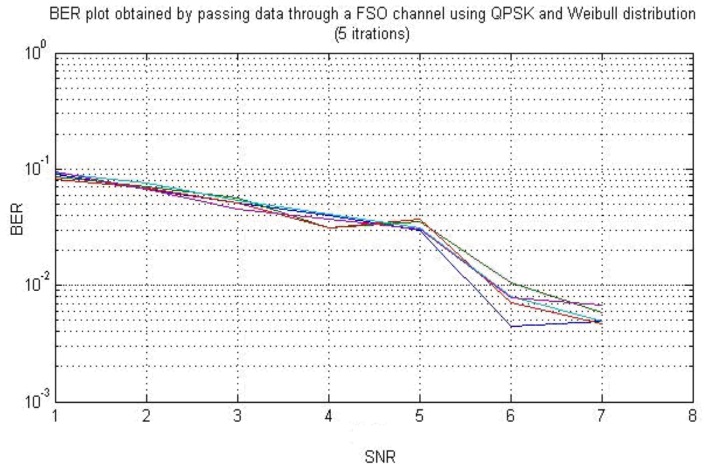

Figure 8. BER plot of LT encoded QPSK modulated data through channel modeled using Weibull distribution

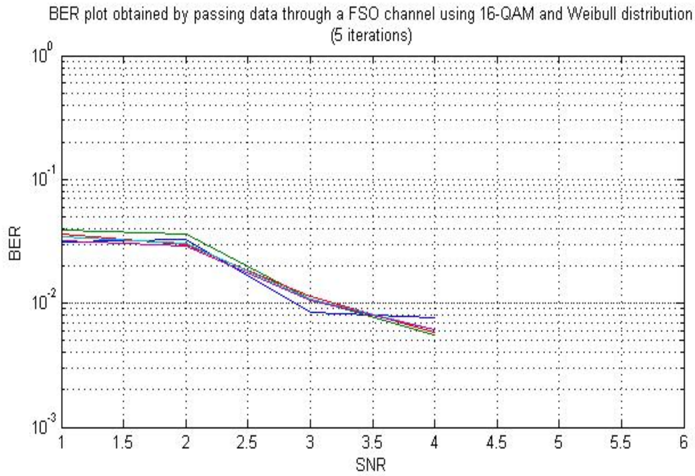

Figure 9. BER plot of LT encoded 16-QAM modulated data through channel modeled using Weibull distribution

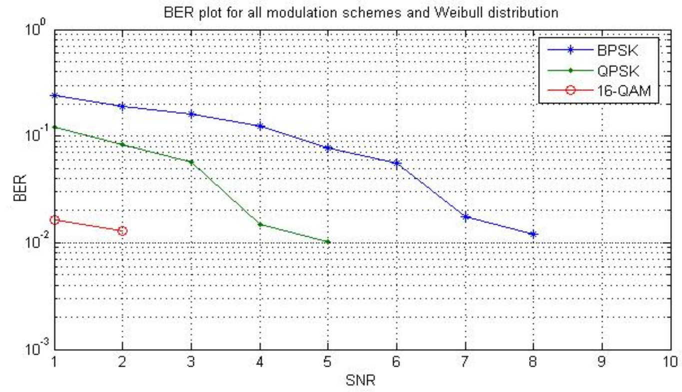

Figure 10. Comparison of BER performance of LT encoded BPSK, QPSK and 16-QAM through channel modeled using Weibull distribution

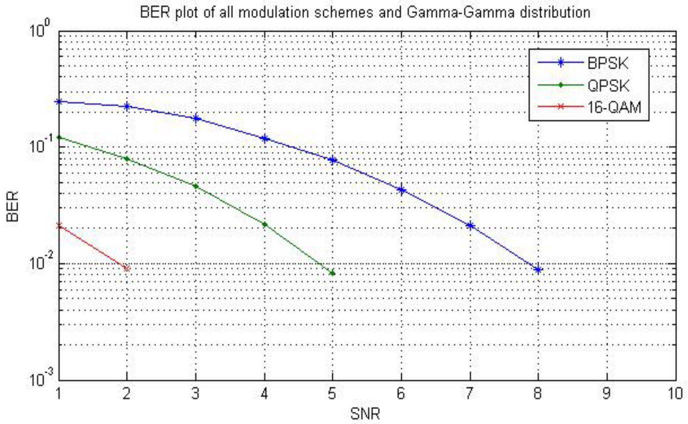

Figure 11. Comparison of different modulation schemes for LT encoding and Gamma- Gamma Distribution

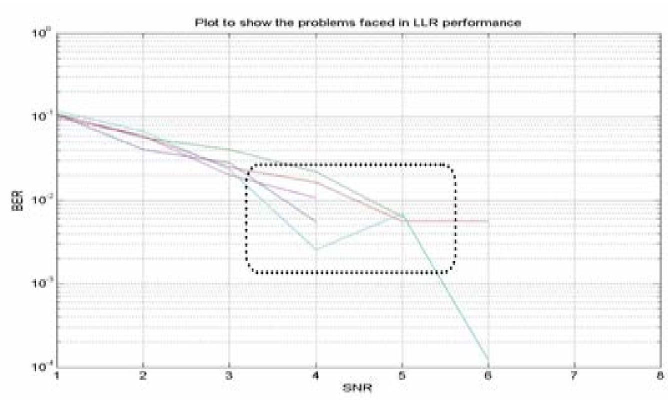

Figure 12. Plot showing abnormality in the LLR performance

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