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Multi-agent search strategy based on centroidal Voronoi configuration

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***Abstract*— We propose a *combined deploy and search* strategy for multi-agent systems using Voronoi partition. Agents such as mobile robots (AGVs, UAVs, or USVs) search the space to acquire knowledge about the space. Lack of information about the search space is modeled as an uncertainty density distribution, which is known *a priori* to all the agents at the beginning of search. It is shown that when the agents are located at the centroid of Voronoi cells, computed with the perceived uncertainty density, reduction in uncertainty density is maximized. While moving toward this optimal configuration, the agents simultaneously perform search acquiring the information about the search space, thereby reducing the uncertainty density. The proposed search strategy is guaranteed to reduce the average uncertainty density to any arbitrary level. Simulation experiments are carried out to validate the proposed search strategy and compare its performance with *sequential deploy and search* strategy proposed in the literature. The simulation results indicate that the proposed strategy performs better than *sequential deploy and search* in terms of faster search, and smoother and shorter robot trajectories.**

I. INTRODUCTION

Autonomous agents are increasingly being used in executing various tasks. There are two possible ways to accomplish complex tasks. One is to increase the cognitive complexity of the agent and other is to use relatively simple, multiple agents, to cooperatively accomplish the given task. Developments in areas such as wireless communication, autonomous vehicular technology, computation, and sensors, facilitate use of large number of agents such as aerial robots (Uninhabited or Unmanned Aerial Vehicles: UAVs), mobile ground robots (Automated Guided Vehicles: AGVs), or mobile surface robots (Uninhabited Surface Vehicles: USVs), equipped with necessary sensors, communication equipment, and computation ability, to cooperatively achieve various tasks in a distributed manner. The motivation for multi-agent systems can be found in nature, where most biological systems such as ants, birds, fish etc., have distributed local decision making capabilities which, in turn, lead to a useful collective behavior such as swarms, flocks, schools, etc. With each agent taking decisions based on only available local information and distributed control law, usually referred to as 'behavior' in biological systems, can lead to coordination among the agents and result in a meaningful collective behavior.

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One class of problems discussed in the literature is that of optimally locating the agents or sensors in the domain of interest which belongs to problem of locational optimization or facility location. A centroidal Voronoi configuration is a standard solution for this class of problems, where the optimal configuration of agents is centroids of corresponding Voronoi cell. Cortes et al. [1] use these concepts to solve a spatially distributed optimal deployment problem for multi-agent systems. A density distribution, as a measure of the probability of occurrence of an event is used, along with a function of the Euclidean distance as measure of how poor the sensing performance is, to formulate the problem. Centroidal Voronoi configuration, with centroid of Voronoi cell computed based on the density distribution within the cell, is shown to be the optimal deployment of sensors minimizing the sensory error. The Voronoi partition becomes natural optimal partition due to monotonic variation of sensor effectiveness function with the Euclidean distance. Lloyd algorithm was used to achieve the optimal configuration. Schwager et al. [2] interpret the density distribution of [1] in a non-probabilistic framework and approximate it by sensor measurement. In [3], [4], a multi-agent search strategy was proposed based on the concept of optimal deployment using Voronoi partition. Here, at each step, the agents first deploy themselves optimally so as to maximize per step reduction in uncertainty density, which is a measure of lack of information about the search space, and then perform search to acquire information.

In this paper we propose a *combined deploy and search* strategy for multi-agent systems based on the search strategy proposed in [4]. Each agent is active within its Voronoi cell ensuring the collision free trajectories. Instead of waiting till the end of deployment step to perform search task as in [4], the agents simultaneously perform search as they move toward the optimal configuration. The paper is organized as follows. In Section II we discuss the problem addressed in this paper, namely, multi-robot search. We preview the *sequential deploy and search* strategy proposed in our previous work [4]. The proposed *combined deploy and search* is presented in Section III. The problem formulation, control law responsible for robot motion and a few implementation issues have been discussed here along with proof of convergence of the CDS strategy. Results and discussions are provided in Section IV and finally the paper is concluded in Section V.

II. MULTI-ROBOT SEARCH

In this section we discuss the problem addressed in this paper. N agents perform search operation in an unknown environment. The search space $Q \subset \mathbb{R}^d$ is a convex polytope in d-dimensional Euclidean space. The lack of information is modeled as an uncertainty density distribution $\phi : Q \mapsto [0,1]$ over Q. The agent configuration at any given time t is $P(t) = (p_1(t), p_2(t), \ldots, p_N(t)) \in Q^N$, with $p_i \neq p_j$, whenever $i \neq j$, where $p_i(t)$ is the position of i-th agent at time t. The sensors' effectiveness is assumed to strictly decrease with the Euclidean distance. The problem addressed in this paper is that of deploying N agents in Q to collect information, thereby reducing the uncertainty density distribution over Q.

During the search operation, sensors gather information about Q, reducing the uncertainty density as,

$$\phi_{n+1}(q) = \phi_n(q) \min_i \{\beta(\|p_i - q\|)\} \tag{1}$$

where, n is "deploy" and "search" count; $\phi_n(q)$ is the uncertainty density at the n-th iteration; $\beta : \mathbb{R} \mapsto (0,1)$ is a function of Euclidian distance of a given point in space from the robot, and acts as the factor of reduction in uncertainty by the sensors; and $p_i = p_i(t)$ is the position of i-th agent at the time instance t when agents gather information at the end of n-th deployment step. At a given $q \in Q$, only the robot with the smallest $\beta(\|p_i - q\|)$, that is, the robot which can reduce the uncertainty by the largest amount, is active. If robots search within their Voronoi cells, then the updating function (1) gets implemented automatically, that is, evaluating the function $\min_i\{\beta(\|p_i - q\|)\}$ is equivalent to evaluating $\beta(\|p_i - q\|)$, with $p_i \in V_i$, the Voronoi cell corresponding to the i-th agent.

In the SDS strategy, the agents get optimally deployed before performing search. In order to maximize the search effectiveness in each search step, following objective function was considered to be maximized.

$$\begin{aligned}\mathscr{H}_n &= \int_Q \Delta\phi_n(q) dQ \\ &= \int_Q (\phi_n(q) - \min_{i \in \{1,2,\ldots,N\}} \{\beta(\|p_i - q\|)\} \phi_n(q)) dQ \\ &= \sum_{i \in \{1,2,\ldots,N\}} \int_{V_i} \phi_n(q)(1 - \beta(\|p_i - q\|)) dQ\end{aligned} \tag{2}$$

The search effectiveness function $\beta : \mathbb{R} \mapsto (0,1)$ is a strictly increasing function capturing effectiveness of the sensor. Consider

$$\beta(r) = 1 - ke^{-\alpha r^2}, \quad k \in (0,1) \quad \text{and } \alpha > 0$$

Here, $ke^{-\alpha r^2}$ represents the effectiveness of the sensor which is maximum at $r = 0$ and tends to zero as $r \to \infty$ and β is minimum at $r = 0$ (effecting maximum reduction in ϕ) and tends to unity as $r \to \infty$ (change in ϕ reduces to zero as r increases). Most sensors' effectiveness reduces over distance as the signal to noise ratio increases with the distance. Thus β, which is upside down Gaussian, can model a wide variety of sensors with two tunable parameters k and α.

The optimal deployment configuration was shown to be a variation of centroidal Voronoi configuration, where each robot is located at the centroid of its Voronoi cell computed with a density $\tilde{\phi}_n(q) = \phi_n(q)ke^{-\alpha r_i^2}$, which is the density as perceived by the sensor. We have seen that the uncertainty reduction will be maximized in a single step of search, if the agents are located at the centroids of respective Voronoi cells. In SDS, the agents get deployed optimally in this sense before performing search.

Typically search problems do not consider dynamics of search agents, as the focus is more on the effectiveness of search, that is, being able to identify region of high uncertainty and distribute search effort to reduce uncertainty. Moreover, it is usually assumed that the search region is large compared to the physical size of the agent or the area needed for the agent to maneuver. In this paper, we assume that the agents are modeled as simple first order dynamical systems as

$$\dot{p}_i = u_i \tag{3}$$

Consider the control law

$$u_i = -k_{prop}(p_i - \tilde{C}_{V_i}) \tag{4}$$

Control law (4) makes the agents move toward $\tilde{C}_{V_i}$ for positive control gain, k_{prop}. We have shown in [4], using LaSalle's invariance principle, that the trajectories of the agents governed by the control law (4), starting from any initial condition $P(0) \in Q^N$, will asymptotically converge to the critical points of $\mathscr{H}_n$.

III. Combined deploy and search (CDS) strategy

In the SDS strategy, The robots first get optimally deployed and then perform search. The "deploy" and "search" steps continue sequentially till the uncertainty density is reduced below a desired value. Here the optimal deployment strategy ensures that the uncertainty density reduction is maximized in each search step. But it does not guarantee optimal trajectories of the robot. During the deployment stage, the robots move without utilizing the sensors. Intuitively, it seems that the trajectories will be closer to optimal if, as the robots are moving toward the respective $\tilde{C}_{V_i}$, they also simultaneously perform the search operation in discrete steps. We define the *latency*, t_s, of the robots as the maximum time taken to acquire the information, process it, and successfully update the uncertainty density. The time interval between each search should be more than t_s. Here we formulate such a strategy and name it *combined deploy and search* (CDS) strategy.

A. Density update

Let the index n represent the intermediate step at which the search is performed and uncertainty density is updated. Using the uncertainty density update rule (1) discussed earlier we can get,

$$\Delta_n \phi(q) = \phi_{n+1}(q) - \phi_n(q) = \phi_n(q) \min_i (1 - \beta(\| p_i - q \|)) \tag{5}$$

Let,

$$\Phi_n = \int_Q \phi_n(q) dQ \tag{6}$$

Integrating (5) over Q,

$$\Delta\Phi_n = \sum_{i \in \{1,2,\ldots,N\}} \int_{V_i} \phi_n(q)(1 - \beta(\| p_i - q \|)) dQ \tag{7}$$

B. Objective function

The objective function (2), used for SDS strategy [4], is fixed for each deployment step as $\phi_n(q)$ is fixed for the n-th iteration. In CDS, the search task is performed as the robots move. Now an objective function to be maximized in order to maximize the uncertainty reduction at the n-th search step is

$$\mathscr{H}_n = \Delta\Phi_n = \sum_{i\in\{1,2,\ldots,N\}} \int_{V_i} \phi_n(q)(1-\beta(\| p_i - q \|))dQ \quad (8)$$

Note that the above objective function is same as (2) except for the fact that n in this case represents the search step count, whereas in (2) it represents 'deploy and search' step count. For $\beta(r) = 1 - ke^{-\alpha r^2}$, the objective function (8) becomes,

$$\mathscr{H}_n = \sum_{i\in\{1,2,\ldots,N\}} \int_{V_i} \phi_n(q)ke^{-\alpha r_i^2} dQ \quad (9)$$

It can be noted that for a given n, the uncertainty density $\phi_n(q)$ at any $q \in Q$ is constant. The gradient is given as (using generalized Leibniz Theorem [5]),

$$\begin{aligned}\frac{\partial \mathscr{H}_n}{\partial p_i} &= \sum_{i\in\{1,2,\ldots,N\}} \int_{V_i} \phi_n(q)ke^{-\alpha(\|p_i-q\|)^2}(-2\alpha)(p_i-q)dQ \\ &= -2\alpha\tilde{M}_{V_i}(p_i - \tilde{C}_{V_i}) \quad (10)\end{aligned}$$

where $\tilde{M}_{V_i}$ and $\tilde{C}_{V_i}$ are the mass and the centroid of V_i with $\tilde{\phi}_n(q) = \phi_n(q)ke^{-\alpha r_i^2}$ as density, which is the uncertainty density as perceived by the sensor. The critical points are same as those obtained for SDS. But the uncertainty changes in every time step and hence the critical points also change. Hence, the corresponding critical points are only the instantaneous critical points. It should be noted that the above treatment is valid for any strictly increasing continuously differentiable $\beta(\cdot)$, with $\tilde{\phi}(\cdot)$ depending on exact nature of the function $\beta(\cdot)$. We assume first order dynamics for robots as in (3) and use control law (4) for making the robots move toward the respective centroids.

The instantaneous critical points and the gradient (10) are used in control law (4) only to make the robots move toward the instantaneous centroids rather than deploying them optimally. Thus, it is not possible to prove any optimality of deployment and we do not prove the convergence of the trajectories here. In CDS, compared to SDS, robots perform more frequent searches instead of waiting till the optimal deployment maximizing per step uncertainty reduction.

To implement the control law, centroid of each Voronoi cell needs to be computed. The computational overhead of computing the centroid can be reduced at the cost of slower convergence using methods reported in the literature such as random sampling and stochastic approximation [6]. In addition, we discretize the search space into grids while implementing the strategy. This simplifies the computation of the centroid of Voronoi cells. The main focus of this paper is design and demonstration of the multi-robot search strategy and finer issues such as computation complexities are beyond the scope of this paper.

It can be shown that the CDS strategy is spatially distributed over the Delaunay graph $\mathscr{G}_D$. Here by spatially distributed we mean that information from neighboring robots is sufficient for computation of control input. A Delaunay graph $\mathscr{G}_D$ is an undirected graph, where two agents/robots are said to be neighbors (connected by an edge) if the corresponding Voronoi cells have non-null intersection. This implies that all the robots need to do computations based on only local information, that is, by the knowledge about position of neighboring robots. Also, the robots should have access to the updated uncertainty map within their Voronoi cells. This can be achieved in several ways. One such way is that all the robots communicate with a central information provider. But it is not necessary to have this global information. The i-th robot can communicate with its Voronoi neighbors $((\mathscr{N}_{\mathscr{G}}(i))$ and obtain the updated uncertainty information in a region $\cup_{\mathscr{N}_{\mathscr{G}}(i)} V_i$. As the Voronoi partition $\{V_i\}$ depends at least continuously on P, the robot configuration [1], in an evolving Delaunay graph, the communication within the neighbors is sufficient for each robot to obtain the uncertainty within its new Voronoi cell. The issues related to communication of uncertainty information are not addressed in the paper except to assume that uncertainty information is available to the robots. It is also possible that the robots can estimate the uncertainty map as done in [2].

Theorem 1: The CDS strategy can reduce the average uncertainty to any arbitrarily small value in finite time.

Proof. Consider the uncertainty density update law (1) for any $q \in Q$,

$$\phi_n(q) = (1 - ke^{-\alpha r_i^2})\phi_{n-1}(q) = \gamma_{n-1}\phi_{n-1}(q) \quad (11)$$

where, r_i is the distance of point $q \in Q$ from the i-th robot, such that $q \in V_i$, the Voronoi cell corresponding to it and, $\gamma_{n-1} = (1 - ke^{-\alpha r_i^2})$.

Applying the above update rule recursively, we have,

$$\phi_n(q) = \gamma_{n-1}\gamma_{n-2}\ldots\gamma_1\gamma_0\phi_0(q) \quad (12)$$

Let $D(Q) := \max_{p,q\in Q}(\| p - q \|)$. We note that

(i) $0 < k < 1$

(ii) $0 \leq r_i \leq D(Q)$. $D(Q)$ is bounded as the set Q is bounded.

(iii) $0 \leq \gamma_j \leq 1 - ke^{-\alpha\{D(Q)^2\}} = l$ (say), $j \in \mathbb{N}$; and $l < 1$

Now consider the sequence $\{\Gamma\}$,

$$\Gamma_n = \gamma_n\gamma_{n-1}\ldots\gamma_1\gamma_0 \leq l^{n+1}$$

Taking limits as $n \to \infty$,

$$\lim_{n\to\infty} \Gamma_n \leq \lim_{n\to\infty} l^{n+1} = 0$$

Thus,

$$\lim_{n\to\infty} \phi_n(q) = \lim_{n\to\infty} \Gamma_{n-1}\phi_0(q) = 0$$

As the uncertainty density ϕ vanishes at each point $q \in Q$ in the limit, the average uncertainty density over Q is bound to vanish as $n \to \infty$. Thus, the average uncertainty density can be reduced to arbitrarily small value in finite time. □

It can be observed that the above proof does not depend on the control law. The theorem depends only on the outcome of the choice of the updating function (1), along with the fact that there is no sensor range limitation, and that the search space Q is bounded. In addition, the theorem does not address the issue of optimality of the strategy which, in fact, depends on the control law which is responsible for the motion of the robots. Further, unlike SDS, maximal uncertainty reduction is also not guaranteed in each search step. The uncertainty reduction in n-th search step is given by $\mathscr{H}_n^* = \sum_i \int_{V_i} \phi_n(q) k e^{-\alpha(\|p_i - q\|)^2} dQ$, where it is not required that $p_i = \tilde{C}_{V_i}$ while performing search. Though the uncertainty reduction in a given search step n in CDS is less than that in SDS, as will be seen in later sections, the CDS performs better compared to SDS in terms of faster uncertainty reduction due to more frequent searches.

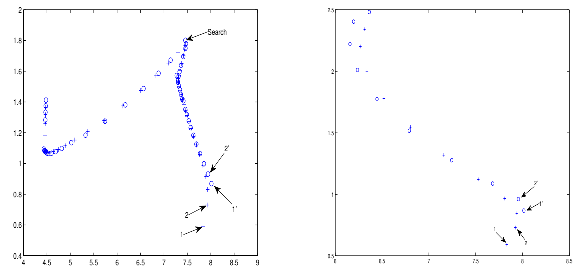
Further, in practical conditions, the robots can communicate with other robots only when they are within the sensor range. The Delaunay graph does not allow sensor range limitations to be incorporated. We need to use *r-Delaunay* graph $\mathscr{G}_{LD}$ to incorporate the sensor range limitations. The scenario changes with incorporation of sensor range limitations into the search strategies. In such a scenario each robot restricts its activity to the region $(V_i \cap \bar{B}(p_i, R))$, which is the Voronoi cell accessible to sensor of i-th robot/agent. Here $\bar{B}(p_i, R)$ is the closed ball of radius R, the sensor range, centered at p_i. The updating of uncertainty density will be within this region and the centroid that is computed will also be within the new restricted area. Though in presence of limit on sensor range, convergence of the search strategies can not be guaranteed, we will show using simulations in later section that with nominal limit on sensor range, the search strategies do reduce the uncertainty below desired level. In addition, it is easy to see that the CDS strategy works well even in presence of constant speed constraint and limit on maximum speed of the robots/agents [7].

It can also be noted that the CDS operates in a synchronous manner by design. If all the robots start at the same instant of time and have synchronized clocks, the search task is performed by every robot after the same interval of time. Given an accurate global clock, synchronization is not a major issue in case of CDS. Further, in [1], authors provide an asynchronous implementation for coverage control which can be suitably modified for CDS to operate asynchronously.

We use Voronoi partition in formulating the search strategies which along with advantages can cause some computation overhead. This issue has been addressed in the literature (see [1] and references therein) and there are a few algorithms that efficiently implement Voronoi partition related computations. Also, Voronoi based strategies result in collision free trajectories in a natural way, which is an added advantage.

IV. Results and Discussion

In this section we show some simulation results to illustrate and validate the CDS strategy in comparison with SDS. The simulation experiments were carried out using MATLAB. The parameters used for these simulations were, Q is a square area in $\mathbb{R}^2$ with axes range of 0-10 units; Initial uncertainty density is a constant distribution of 0.75 over Q; A saturation on the speed of agents was fixed at 1 unit; $k_{prop} = 0.5$; We use $\beta(r) = 1 - ke^{-\alpha r^2}$, with $k = 0.8$ and $\alpha = 0.1$; The iterations were terminated when the maximum density over Q reached below 0.05. A discrete implementation of the control law (4) is used with time period of 1 unit.



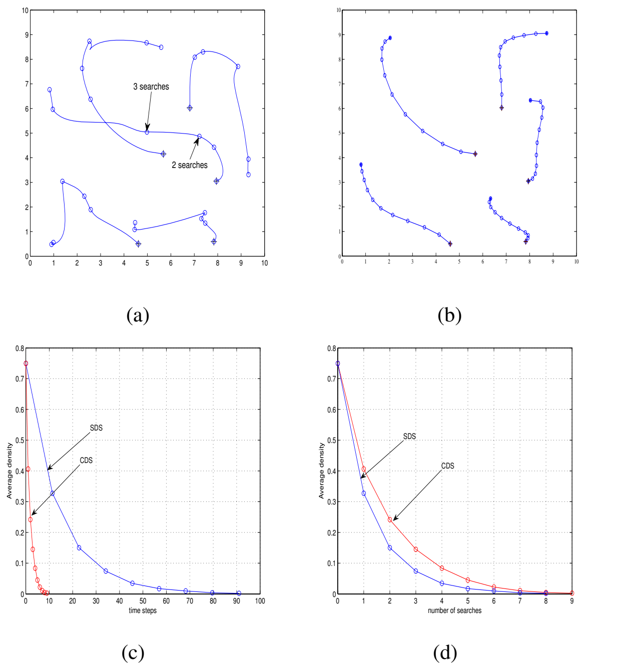
(a) (b)

Fig. 1. Illustration of the process of a robot following the respective centroid (a) with SDS and (b) with CDS. In both cases the robot location in each time step is shown by '+', and corresponding centroid locations are shown by 'o'.

Figure 1(a) illustrates 'robot 2' moving toward centroid corresponding to its Voronoi cell with SDS strategy. Robot's positions are marked with '+' while 'o' marks the centroids at successive time instances. Positions of robot in first two time steps are marked as 1 and 2, while those of centroids marked with $1'$ and $2'$. It can be observed that the robot is tracking the centroid, which is changing as the Voronoi cell is changing. Deployment stops and search is performed when the robot is sufficiently close to the corresponding centroid. One of the search instances is also marked, where, after search, in order to track the next centroid, the robot takes an abrupt turn. This leads to a non-smooth trajectory. Figure 1 (b) illustrates one of the robots moving toward centroid corresponding to its Voronoi cell with CDS strategy. It can be observed that the robot is tracking the centroid, which is changing as the Voronoi cell and the uncertainty density are changing.

Figures 2 (a) and (b) compare the trajectories of robots with SDS and CDS strategies with 5 robots without any limit on sensor range. The trajectories with CDS are much smoother and shorter. The instances of search are indicated by 'o' along the trajectories. It can be seen that the search is performed at every discrete step in CDS, whereas the search is performed only after each optimal deployment SDS. Though there are 8 "deploy and search" steps in SDS, only 5 'o's are visible. In two of steps, multiple searches have been performed as the centroids in successive steps were closer than some tolerance limit $d_{tol} = 0.3$. Thus, there was no movement in corresponding deployment step.

Figure 2(c) compares the history of uncertainty density of SDS and CDS, and it can be observed that the CDS reduces uncertainty relatively faster than SDS, in terms of number of time steps. Figure 2 (d) shows the reduction in average

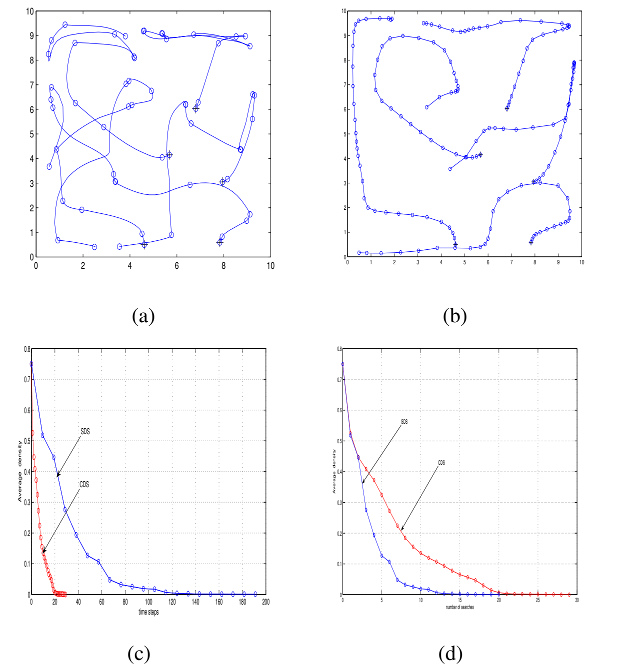


(a) (b) (c) (d)

Fig. 2. Trajectories of robots with N=5 and without limit on the sensor range for (a) SDS strategy and (b) CDS strategy. In both cases the points marked '+' indicate the starting locations of robots and 'o' indicate the end of deployment and points in space where search is being performed. In SDS, at a few places the search is performed more than once. This is indicated in trajectory of 'robot 1'. The reduction in average uncertainty density is shown in (c) against the number of time steps and (d) against the number of searches, for SDS and CDS. Even in (c) and (d), 'o' indicate the search instances.

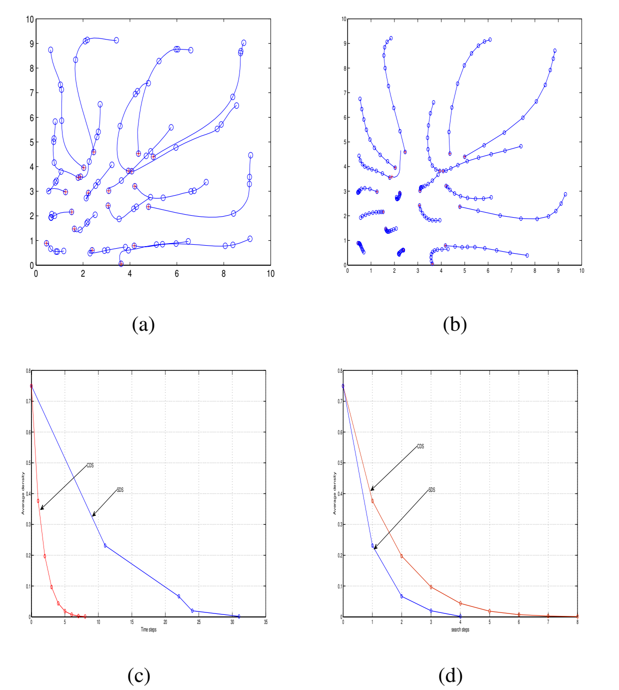
uncertainty density with number of searches for SDS and CDS. It can be observed from this figure that SDS reduces the uncertainty in relatively fewer steps. This is apparent by very concept of optimal deployment in SDS. CDS takes about 4 searches to reduce uncertainty below 0.1, whereas SDS does this in only 3 searches. If SDS requires over 30 time steps to achieve this reduction, CDS needs 4 time steps. We can observe a tradeoff between the number of searches and and number of time steps required to accomplish in CDS and SDS. Once the uncertainty reduces to a large extent in initial search steps, by nature of the uncertainty density update rule (1), amount of reduction in subsequent searches is less in both SDS and CDS.

Figures 2, 3, 4, and 5 show the trajectories for N=5 and N=20, with and without sensor range limitations along with the plot of average uncertainty density for both strategies. Again, all the simulation results support the fact that the CDS strategy leads to much shorter and smoother trajectories than those with SDS strategy with the same parameters as discussed before. These results also indicate that the CDS strategy is more effective compared to the SDS strategy for similar conditions in the sense of faster reduction in uncertainty density. This is due to increased frequency of searches in case of CDS compared to that of SDS. However, SDS is more effective in terms of requiring fewer search steps.



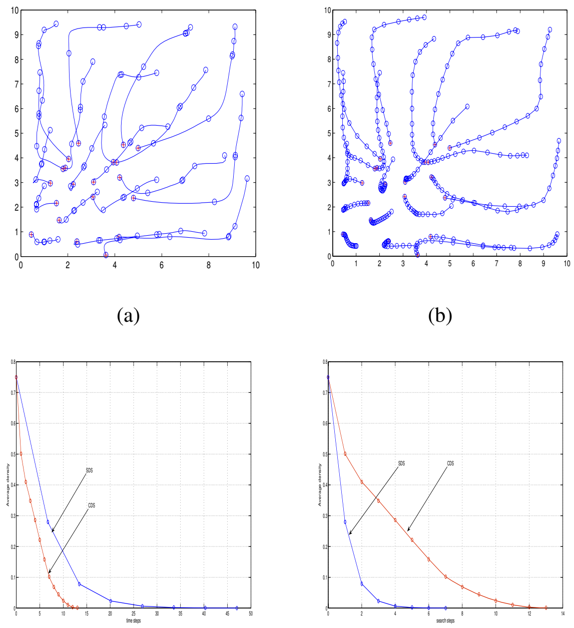
(a) (b) (c) (d)

Fig. 3. Trajectories of agents with N=5 and with sensor range limit of 2 units (a) SDS strategy and (b) CDS strategy. In both cases the points marked '+' indicate the starting locations of robots and 'o' indicate the end of deployment and points in space where search is being performed. The reduction in average uncertainty density is shown in (c) against the number of time steps and (d) against the number of searches, for SDS and CDS. Even in (c) and (d), 'o' indicate the search instances.



(a) (b) (c) (d)

Fig. 4. Trajectories of agents with N=20 without limit on sensor range for (a) SDS strategy and (b) CDS strategy. In both cases the points marked '+' indicate the starting locations of robots and 'o' indicate the end of deployment and points in space where search is being performed. The reduction in average uncertainty density is shown in (c) against the number of time steps and (d) against the number of searches, for SDS and CDS. Even in (c) and (d), 'o' indicate the search instances.



(a) (b)

(c) (d)

Fig. 5. Trajectories of agents with N=20 and with sensor range limit of 2 units for (a) SDS strategy and (b) CDS strategy. In both cases the points marked '+' indicate the starting locations of robots and 'o' indicate the end of deployment and points in space where search is being performed. The reduction in average uncertainty density is shown in (c) against the number of time steps and (d) against the number of searches, for SDS and CDS. Even in (c) and (d), 'o' indicate the search instances.

Comparing Figure 2 with Figure 3, and Figure 4 with Figures 5, it can be observed that limit on the sensor range results in longer agent trajectories in both the search strategies. This is due to reduced coverage by sensors due to the range limits.

If we compare the simulation results with $N = 5$ (Figures 2, 3) with those with $N = 20$ (Figure 4 and 5), it can be observed that for SDS strategy, an increase in number of agents leads to smoother and somewhat shorter agent trajectories, though total trajectory length for all the agents is higher for the latter case. With CDS, individual agent trajectory lengths are comparable, and hence less number of agents lead to a shorter total trajectory length.

Comparing Figures 2 and 4, it can be observed that, without the sensor range limits, with CDS, agent trajectories do not intersect each other or themselves, for both $N = 5$ and $N = 20$ cases; whereas SDS with $N = 5$, lead to intersecting agent trajectories. For $N = 20$, even SDS strategy leads to less intersections among trajectories. When the sensor range limits are imposed (see Figures 3 and 5), the instances of intersection of trajectories increase in both strategies, though it is less prominent in the case of CDS. In a search task, intuitively, one may feel that it is not desirable that an agent intersects its own trajectory, or intersects those of other agents, as it may lead to duplication of search effort. However, it should be noted, that the search operation presented here differs from the coverage problems such as Lawn-Mower's search were each point in space needs to be visited by any agent once in order to gather information at that point. In our problem formulation, uncertainty will not become zero once an agent passes through a point in the search space. Only repeated search at a point makes the uncertainty tend to zero. Such a paradigm is common in search literature. Thus, it is possible that an optimal strategy will make an agent pass through a point in the search space more than once. In all cases (Figures 2, 3, 4, and 5) it can be observed that the agents move away from each other and cover the search space.

We have formulated the problem and shown that the CDS strategy reduces the uncertainty density when no limit is imposed on the sensor range. The simulation results indicate that both strategies reduce the uncertainty density below the desired level even with nominal limit on sensor range.

V. Conclusions

In this paper we proposed a CDS strategy for multiple agents such as mobile robots to acquire information about a space. We have shown that the centroidal Voronoi configuration with respect to the density as perceived by the sensors are the instantaneous critical points of the objective function. A control law was proposed, which moves the agents toward the respective centroids. In this strategy, as the agents move toward the centroids, they simultaneously perform search. It has been observed that the strategy is spatially distributed over the Delaunay graph. We have proved that the CDS strategy is able to reduce the average uncertainty density to arbitrarily low level when no limit is imposed on the sensor range.

Simulation experiments were carried out for different conditions and results of these experiments were discussed. The simulation results indicated that the proposed CDS strategy performs quite well even in presence of sensor range limit, and leads to shorter and smother robot trajectories than those of the SDS strategy with the same parameters.

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