Estimation of degradation of performance of optical network due to crosstalk

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Abstract—**The aim of this paper is to carry out an analysis of optical network by calculating bit error rate (BER) as a function of signal to noise ratio by using Hermitian polynomial for different homodyne crosstalk levels. The results have been plotted for different crosstalk level for BER as a function of single to noise ratio (SNR) by using exact analysis and Gaussian approximation and the results are compared. Simulations have been carried out in MAT Lab Version 7.4. We present an exact analytical probability density function and a closed form bit error rate formula for optical network which performance is degraded by homodyne crosstalk. In homodyne (intrachannel) crosstalk, the interfering signal is at same wavelength as the desired signal. This effect is more severe than interchannel crosstalk (in which interfering signal operates at different wavelength), since the interference falls completely within the receiver bandwidth.**

Index terms—**homodyne crosstalk. Optical networks, wave length division multiplexing**

I. INTRODUCTION

Optical networks are high-capacity telecommunications networks based on optical technologies and components that provide routing, grooming, and restoration at the wavelength level as well as wavelength-based services [1]. An optical network connects computers (or any other device which can generate or store data in electronic form) using optical fibers. Optical networks seem to be the medicine for all problems in long-haul and metro networking. Fiber optics are “pretty clear”, but not perfectly clear. There can be impurities and construction limitations that will constrain the optical transmission properties. The cladding does not absorb any light from the core, the light wave can travel great distances. However, some of the light signal degrades within the fiber, mostly due to impurities in the glass. As optical fiber data rates, transmission lengths, number of wavelengths and optical power levels have increased, non-linearity effects arose[2]. Non-linear effects of fiber became apparent with specialized applications such as undersea installations. Some of these effects are important to know when designing fiber optics systems, include: stimulated Brillouin scattering (SBS), stimulated Raman scattering (SRS), four wave mixing (FWM), self-phase modulation (SPM), cross-phase modulation (XPM), and intermodulation (mixing).

Non-linear effects limit the amount of data that can be transmitted on a single optic fiber. System designers must be aware of limitations and the steps that can be taken to minimize the detrimental effects of fiber non-linearties. Dense wave length division multiplexing (DWDM) transmission in which individual wavelength channels are modulated at rates of 10 Gb/s offers capacities of $N \times 10$ Gb/s, where N is the number of wavelengths. To transmit such high capacities over long distances require operation in the 1550 nm window of dispersion shifted fiber. To preserve an SNR, a 10 Gb/s system operating over long distances and having nominal optical repeater spacing of 100 Km needs optical launch power of around 1 mW per channel. For such WDM[5] system, the simultaneous requirement of high launch power and low dispersion give rise to the generation of new frequencies due to four wave mixing. Four wave mixing is third order nonlinearity in silica fiber than is analogous to intermediation distortion in electrical Systems. When wavelength channels are located near the zero-dispersion point, three optical frequencies (v_i,v_j,v_k) will mix to produce a forth intermodulation product v_{ijk} given by

$$V_{ijk} = v_i + v_j - v_k \quad \text{with } i, j \neq k \qquad (1)$$

When this new frequency falls in the transmission window of the original frequencies, it can cause severe crosstalk [3]. Cross talk can be introduced by any component in a WDM system, including optical filter, wavelength multiplexer and demultiplexer, optical switches, optical amplifiers and fiber itself.

II. TYPES OF CROSSTALK

The two type of crosstalk that can occur in WDM systems are interchannel crosstalk[11] and intrachannel (homodyne) crosstalk[4]. Both of these cause penalties in the system performance.

A. Interchannel Crosstalk

Interchannel crosstalk arises when interfering signal comes from a neighboring channel that operates at different wavelength. This occurs when wavelength selecting device imperfectly rejects or isolates the signals from other nearby wavelength channel. Crosstalk then arises since these spurious neighboring signals could fall partially within the receiver paasband.

B. Intrachannel (Homodyne) Crosstalk

For intrachannel (homodyne) crosstalk, the interfering signal is at same wavelength as the desired signal. This effect is more severe than interchannel crosstalk, since the interference falls completely within the receiver bandwidth. Even for a single interferer, analysis of homodyne crosstalk was largely based on Gaussian approximation, though there were evidences that this assumption is incorrect. Representing a worst-case assumption and serving well for conservative system design, the Gaussian approximation is only valid for a large number of more or less the same intensity and statistically independent interferers. Therefore, a non-Gaussian model may better estimate the system performance. Furthermore, a non-Gaussian model can be used to verify the condition of the validity of the Gaussian approximation.

Non-Gaussian model had been developed for a single interferer by series[7]-[9] expansion and numerical integration or simulation. However, most of the methods of require complicated numerical calculation to evaluate the system performance.

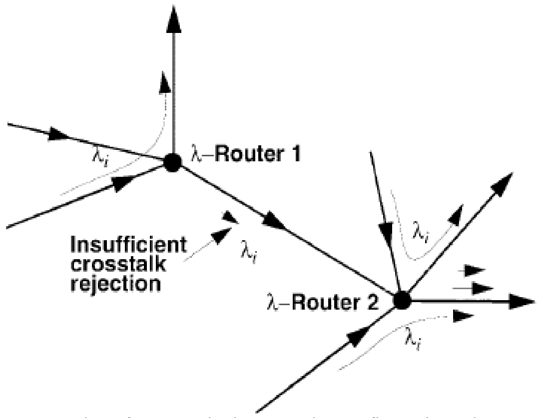

Fig. 1. Example of an optical network configuration that may induce homodyne cross-talk interference.

III. ANALYSIS OF PERFORMANCE DEGRADATION IN OPTICAL NETWORK BY HOMODYNE CROSSTALK

Homodyne crosstalk [3]-[6] may be originated by many different sources. Fig. 1 shows a configuration of an optical network that may induce homodyne crosstalk with similar or identical wavelength to the signal wavelength. While the channel at wavelength at the input of wavelength router 1 should not appear at the input of wavelength router 2, due to insufficient crosstalk rejection in router 1, small amount of crosstalk appears at the input of wavelength router 2 as homodyne crosstalk. Alternatively, two input signals having the same wavelength may appear at different input ports of the same wavelength router (router 2). There is no homodyne crosstalk in ideal case because two signals are routed to different output ports. However, any leaking or insufficient isolation may induce homodyne crosstalk. Those homodyne crosstalks beat with the signal and severely degrade the system performance.

Assuming an optical signal without modulation, the electrical intensity of the desired optical signal is

$$E_i(t) = E_i e^{-j\omega_i t - j\varphi_o(t)} \quad (2)$$

where $E_i(t)$ is the intensity of the signal electric field, ω_i is the angular frequency of the optical signal, $\varphi_0(t)$ is the random phase due to laser phase fluctuation and a small homodyne crosstalk originating from a neighboring node is

$$E_{i,x}(t) = \sqrt{x} E_i e^{-j\omega_i t - \varphi_1(t)} \quad (3)$$

where x is the crosstalk level in optical power, φ_1 are the random phases due to laser phase fluctuation Without loss of generality, for a unit detector responsivity and worst-case assumption of identical polarization of signal and crosstalks, the photocurrent is ignoring the small terms in the order of x first, the overall receiver noise in the photodetector is

$$i(t) = \left|E_i + \sqrt{x}\, E_i e^{-j\varphi(t)}\right|^2 \quad (4)$$

where $\varphi(t) = \varphi_1(t) - \varphi_0(t)$ is also a random phase Ignoring the small terms in the order of x first, the overall receiver noise in the photodetector is

$$n(t) = A\cos[\varphi(t)] + n_0(t) \quad (5)$$

where $A = 2\sqrt{x}E_i^2$ is the crosstalk amplitudes, $n_0(t)$ = the usual Gaussian noise in the receiver In each homodyne crosstalk beating, for a random phase of ϕ (t), the pdf of A $\cos[\phi(t)]$ is given by

$$p(x) = \frac{1}{\pi}(A^2 - x^2)^{1/2} \quad \text{for } -A < x < +A \quad (6)$$

which yields the characteristic function of $J_0(A\omega)$, where $J_0(.)$ is the zero-order Bessel function[7] of first-kind.

$$J_0(A\omega) = 1 - \frac{(A\omega)^2}{2^2} + \frac{(A\omega)^4}{2^2 4^2} - \frac{(A\omega)^6}{2^2 4^2 6^2} + \cdots + \infty \quad (7)$$

and characteristic function[8] (discrete case) of random variable X is defined by

$$\psi_X(\omega) = E(e^{j\omega X}) = \sum_i e^{j\omega X_i} p_X(X_i) \quad (8)$$

If $\psi_X(\omega)$ is known , probability density function[8] $p_X(x)$ can be found from the inverse fourier

$$p_X(x) = \frac{1}{2\pi}\sum_{k=0}^{\infty} \psi_X(\omega)\, e^{j\omega x_k} \quad (9)$$

By using above formulas, the characteristic equation of n(t) given by as

$$\psi_n(\omega) = J_0(A\omega)\exp\left(\frac{-\sigma^2\omega^2}{2}\right) \quad (10)$$

where σ_0^2 and $\exp(-\sigma_0^2\omega^2/2)$ are the variance and characteristic function of the receiver Gaussian noise $n_g(t)$, respectively. In the following section, the BER[3]-[6] due to the summation of homodyne crosstalk and Gaussian noise is evaluated according to the characteristic function of the overall noise. Taking the series expansion and found the inverse transform of the characteristic function, the probability density function (pdf) of n(t) is

$$p_n(r)\frac{1}{\sqrt{2\pi}\sigma} \times \sum_{k=3}^{\infty}\frac{(A^2/2\sigma^2)^k c_k}{2^k (k!)^2} H_{2k}\left(\frac{r}{\sigma}\right) exp\left(-\frac{r^2}{2\sigma^2}\right) \quad (11)$$

where $A^2/2\sigma^2$ is the ratio of crosstalk to Gaussian variance, an $H_n(x)$ *is a* Hermitian polynomial of order nth given by

$$H_n(x) = (-1)^n e^{\frac{x^2}{2}} \frac{d^n e^{-x^2/2}}{dx^n} \quad (12)$$

Assuming a detection level of d, the error probability or the cumulative tail probability [8] is

$$p_b = \int_{-\infty}^{-d} p_{n1}(x)dx = \frac{1}{2}\operatorname{erfc}\left(\frac{d}{\sqrt{2\sigma}}\right)$$
$$+\frac{1}{\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{(A^2/2\sigma^2)^k}{2^K (k!)^2} H_{2k-1}\left(\frac{d}{\sigma}\right)\exp\left(-\frac{d^2}{2\sigma^2}\right) \quad (13)$$

The BER at the receiver can be evaluated according to the error probability p_b.

The BER of the system is $Q(\rho_G)$ for an optical communication system[8] contaminated by only gaussian noise, where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-x^2/2}dx$ is the cumulative tail of the normalized Gaussian distribution, $\rho_G = I_1 - I_0/(\sigma_1 - \sigma_0)$ is the Q factor or signal to noise ratio of the system and $I_1, I_0, \sigma_1^2, \sigma_0^2$ are the photocurrent and the noise variance at

ONE and ZERO levels, respectively. For an infinite extinction ratio (or I_1/I_0 Larger than 20 db), and identical ZERO and ONE level Gaussian noise, the threshold level $d = I_1/2$ and $\sigma = \sigma_0 = \sigma_1$, the Q-factor $\rho_G = d/\sigma$. $A^2/2\sigma^2$ is equal to the ratio of crosstalk to gaussian noise. The homodyne crosstalk occurs only if both the signal and crosstalk channel are transmitted in ONE levels, or A= $2\sqrt{2d}\sqrt{2dx}$. After some algebra, we can find that $A^2/2\sigma^2 = 8\,\rho_G^2\,x$. The threshold at the middle of the "eye" is $(1+x)I_1/2$. Considering all combination of ONE and ZERO levels of the signal and crosstalk, with the term of x also taking into account, closed form formula of BER [8,9] is

$$BER = \frac{1}{2}Q[\rho_G(1-x)] + \frac{1}{2}Q[\rho_G(1+x)]$$
$$\frac{1}{4\sqrt{2\pi}}\sum_{k=1}^{\infty}\frac{2^{2k}\rho_G^{2k}x^k}{(k!)^2}H_{2k-1}[\rho_G(1+x)]\exp\left[-\frac{\rho_G^2(1+x)^2}{2}\right] \quad (14)$$

Where the first two terms are contributed by Gaussian noise, the third term is contributed by the non-Gaussian nature of homodyne crosstalk. The first term is for signal and crosstalk channel in different levels. The second term is the effect of Gaussian noise for both crosstalk and signal channel in the same level. The third term is the effect of crosstalk for both crosstalk and signal channel in ONE level.

IV. Simulation Results and Analysis

The following simulation results provide the performance degradation[5] induced by homodyne crosstalk . The results are based on bit error rate (BER) performance over a range of signal to noise ratio (SNR).With Gaussian assumption, the BER can be approximated by[08],[09]

$$\mathrm{BER} = \frac{1}{2}Q[\rho_G(1-x)] + \frac{1}{4}Q[\rho_G(1+x)] + \frac{1}{4}Q\frac{\rho_G(1+x)}{\sqrt{1+8\rho_G^2 x}} \quad (15)$$

$8\rho_G^2 x$ is the crosstalk to Gaussian noise ratio for both signal and crosstalk channel in ONE levels, the average ratio is $4\rho_G^2 x$.

A. Results Analysis of BER as a Function of SNR of ρ_G (Exact Analysis)

We have simulated the equation 14 in Mat Lab and plotted the graph for BER as a function of SNR (ρ_G) for different crosstalk level by exact analysis (without approximation).

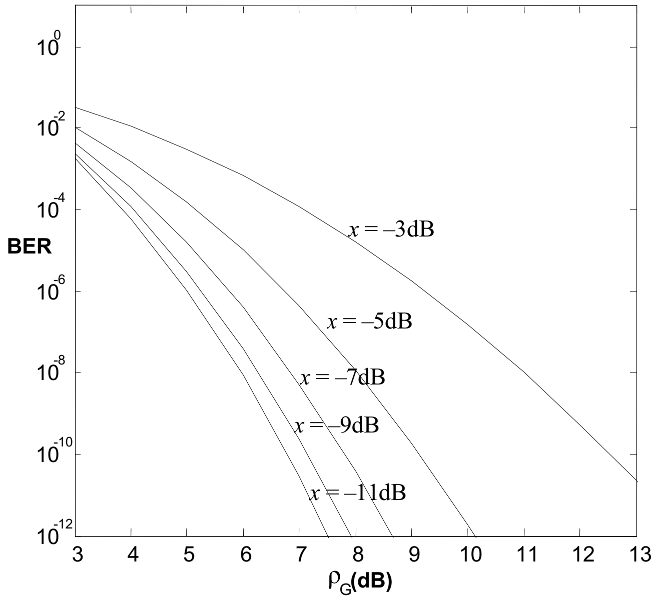

Fig. 2. BER as a function of SNR(ρ_G) for different Crosstalk levels (Exact Analysis)

B. Result Analysis of BER as a Function of SNR of ρ_G (by Gaussian Approximation)

We have simulated the equation 15 in Mat Lab and plotted the graph for BER as a function of SNR (ρ_G) for different crosstalk level by applying gaussian approximation. Above plot shows that gaussian approximation overestimates the degradation induced by homodyne crosstalk.

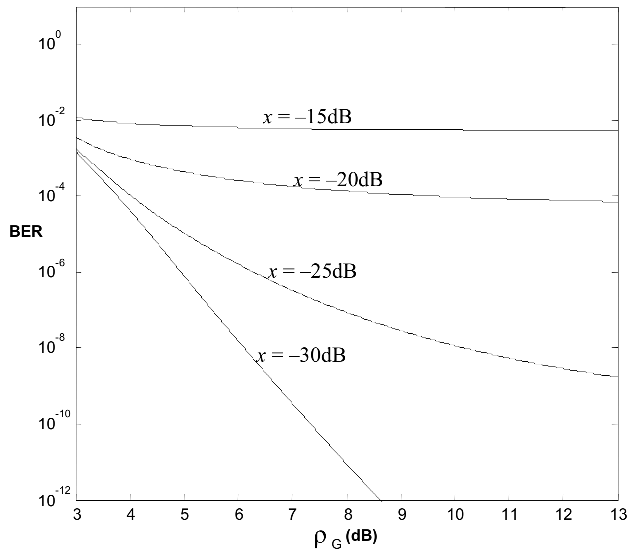

Fig. 3. BER as a function of SNR (ρ_G) for different crosstalk levels

From figure 2 and 3 show that the BER provided by Gaussian approximation is always higher than the BER provided by exact analysis. For example, the Gaussian apprixmation provides a BER floor around 10^{-2} for a crosstalk level of -15 dB but exact analysis shows no BER floor.

C. Result Analysis of BER as a Function of Signal to Overall Noise Ratio of $\rho_G/\sqrt{1+8\rho_G^2 x}$ (by Gaussian Approximation)

We have simulated the equation 15 in Mat Lab and plotted the graph for BER as a function of SNR of $\rho_G/\sqrt{1+8\rho_G^2 x}$ which represents the signal to overall noise ratio.

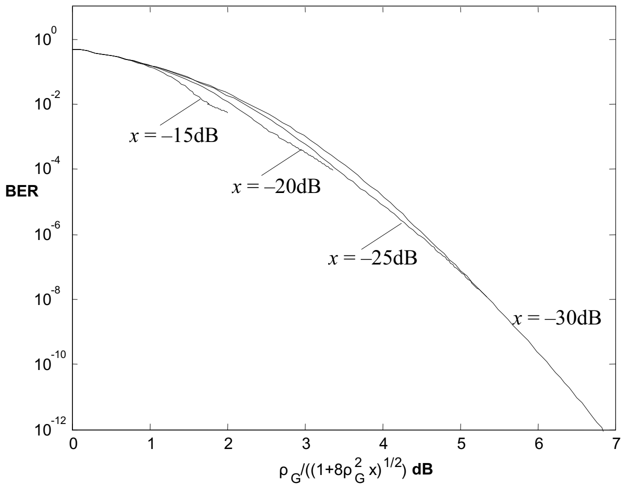

Fig. 4. BER as a function SNR ($\rho_G/\sqrt{1+8\rho_G^2 x}$) for different crosstalk levels

V. CONCLUSION

This paper describes performance degradation in optical Network by homodyne crosstalk. This paper describes how Hermitian polynomial is used to calculate bit error rate induced by homodyne crosstalk. In this paper Analysis of BER as a function of SNR and results in MAT Lab programming are plotted for different crosstalk levels. Results in figure 1, 2, and 3 shows the performance plots for BER provided by Gaussian approximation and exact analysis. These results show that BER provided by Gaussian approximation is always higher than the BER provided by exact analysis.This work also concludes that as the value of crosstalk level is increased the value of bit error rate is increased over same value of signal to noise ratio. Based on these results future research can be carried out. The bit error rate induced by homodyne crosstalk can be calculated by using Hyper-geometric function and new result can be obtained.

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