Stability Analysis of Stratified Soil Slopes by Optimization Technique Using Janbu's Generalized procedure of Slices

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ABSTRACT : The networks to facilitate communications have created large number of cuttings and embankments all along the west coast of India which is subjected to high intensity of rainfall . The heavy rainfall has converted the stable slope to unstable slopes over the years. Hence there is a need to study the effect of rainfall and other factors on the stability of slopes.

Typical stratified slope is considered for the stability analysis and the general computer program is developed in C language which optimizes the factor of safety. The factor of safety is calculated using Janbu's GPS and Davidon-Fletcher-Powel (DFP) technique is used for optimization. The slope with berm or without berm and with or without tension crack are analyzed for stability. The factor of safety decreases with the increase in unit weight, pore pressure ratio, height of slope and slope angle and improves with shear strength parameters and berm width. The program is flexible in varying the input parameters to suit to the field conditions and their effects are presented in this paper.

1 Introduction

Human settlements and their subsequent developmental activities, especially in the urban and semi-urban areas, are drastically changing the landforms and thereby disturbing the original drainage pattern. The changed drainage pattern could be the primary cause for the failure of the soil masses located with in a soil slope. This is a serious problem nowadays, with rapid urbanization especially in hilly and sloping terrain areas. Landslides are relatively rapid down slope soil and rock movements, which take place characteristically on one or more, discrete, bounding slip surfaces, which define the moving mass. Landslides constitute an important geotechnical problem that involve a variety of geomaterials in a variety of geological and climatic contexts, and that have a major socio-economic impact on civilization.

There are large network of roads and railways along the west coast of India. These networks have created large number of artificial slopes all along the coast. These slopes are designed safely for site conditions at the time of construction. The west coast is subjected to high intensity of rainfall of the order of around 5000mm in about three months period. However, there have been change in drainage pattern after the construction of the slopes and excess pore water pressures have been allowed to build up. The cumulative effect of rainfall has converted the stable slope to unstable over the years. Slopes with higher heights very often involve berms in between and stratified soil mass. This paper reports, slope stability in stratified soil mass with various combinations of slope geometry parameters and soil parametrs.

2 Stability Analysis

Over the years, limit equilibrium methods have been extensively refined by various investigators. Perhaps the most remarkable refinement has come in the form of development of methods which do not require any priori assumption regarding the shape of the slip surface. Some of the widely studied methods in this category are those credited to Janbu (1957, 1973), Morgenstern and Price (1965) and Spencer (1967, 1973). Subsequently, the refinement, which has so far been concentrated only on the method of analysis, has been extended to the search for critical slip surface. It is now well appreciated that limit equilibrium slope stability analysis is a problem of optimization wherein the shape and location of the critical slip surface which yields the minimum factor of safety, are found out. The use of powerful and efficient minimization techniques available in the optimization literature has been a topic of increasing interest among the researchers in the area.

Slope stability analysis is essentially a problem of optimization, namely, the determination of the slip surface that yields minimum factor of safety. Slope section can be analyzed using the generalized procedure of slices (GPS) developed by Janbu in conjunction with sequential unconstrained minimization technique given by Davidon-Fletcher-Powell. This method is capable of locating the critical shear surface corresponding to the minimum factor of safety without putting any prior restrictions on the shape of the slip surface. In this technique the stability problem is posed as an optimization problem wherein the factor of safety is minimized with respect to the co-ordinates of the slip surface and thus critical surface is located.

Stability analysis consists of determining for each slope a number, namely its safety factor F. In any slope a number (possibly an infinite number) of potential failure surfaces may be considered. Each of them has its own safety factor. The minimum safety factor among the set of safety factors of the various potential failure surfaces is the safety factor of the slope, and the corresponding surface will be critical sliding surface.

Safety factor F is defined for a given failure mechanism as the ratio between the Resisting Forces (RF) or moments and the Disturbing Forces (DF) or moments.

Safety Factor = Resisting Forces or moments / Disturbing Forces or moments. (1)

3 Cross Section of Slope

Typical stratified slope with three layers is considered for the stability analysis. The general computer program is developed in C language which optimizes factor of safety for the selected stratified slope. The program gives the factor of safety and the cross section of critical slip surface in three types of slope failures as Base, Toe and Slope failure. The slopes with more heights are generally accommodated with berm of varying width in between the slope. The program is modified to take the effect of tension crack existing at the top layer in cohesive soils.

3.1 Slope with absence of berm

The Figure 1 shows the cross section of slope without berm. The angle of slope is taken as α in degrees and H1 as the vertical height of slope. The length of horizontal plane at the top of slope and at the toe of slope is taken as equal to two times the height (H1) of slope. The bottom layer is of thickness HL1= (0.5*H1) with soil parameters cohesion as c1, angle of internal friction as Φ1, and unit weight of soil as γ1.Similarly the middle layer and top layer are of thickness HL2 = (1.0*H1) and HL3 = (2.0*H1) with corresponding soil parameters are considered. Origin is taken as the right extreme bottom which is the bottom most point of layer1. The two co-ordinate axes are taken from origin: along upwards is the Y axis and towards left is the X axis. The (x, y) co-ordinates of slope cross section points are denoted as (Sx [ ], Sy [ ]).

3.2 Slope with presence of berm

The Figure 2 shows the cross section of slope with berm. The bottom layer is of thickness HL1 = 0.25*(HU+HL), the middle layer of thickness HL2 = 0.5*(HU+HL) and top layer of thickness HL3 = 1.25*(HU+HL) with corresponding soil parameters are considered. The angle of slope is taken as αU at above the berm and αL at below the berm in degrees and HU and HL as the vertical height of slope at upper part and lower part respectively. The length of horizontal plane at the top of slope and at the toe of slope is taken as equal to two times the height (HU + HL) of the slope.

4 Formulation of the problem

Figures 1 and 2 shows the geometry of the slope where a general potential slip surface based on type of failure is assumed and the sliding mass is divided into 8 slices. For the given geometry of the slope and soil properties the factor of safety is a function of shape and location of the potential slip surface. The problem is to determine the shape and location of the shear zone that yields the minimum factor of safety. Let X[0], X[1], X[2],……X[6] be the design variables of a general potential slip surface.

The objective function is the factor of safety and an expression for the same can be obtained from the Janbu's method. The factor of safety can be expressed in terms of the design vector as F = f(D) = f(X[0],X[1],X[2],…..X[6]). Find the design vector D such that F = f(D) is minimum subjected to

$$G_j (D) <=0 \quad (2)$$

Where j = 0, 1, 2,…….(M-1), where M is number of constraints. In order to ensure that the slip surface is physically reasonable and acceptable constraints are imposed on the shear surface.

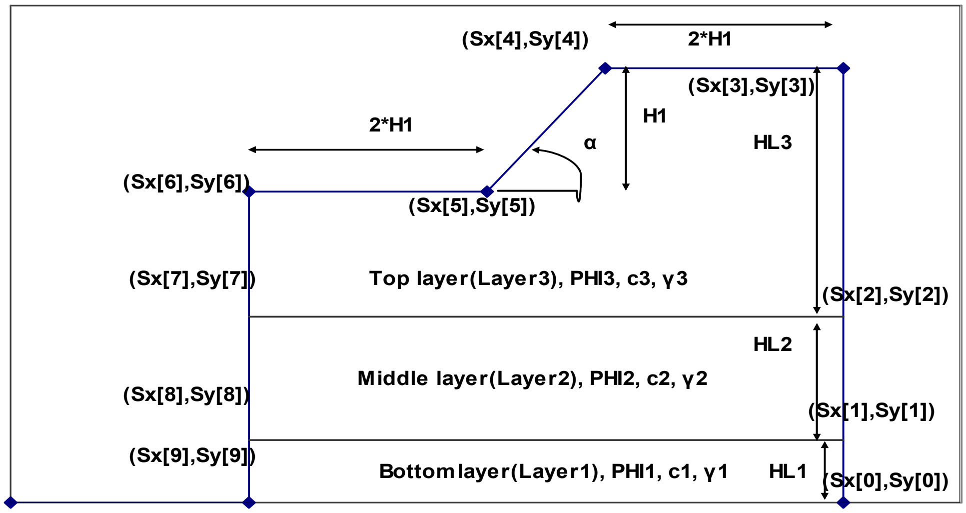

Figure 1. Cross section of slope with absence of berm.

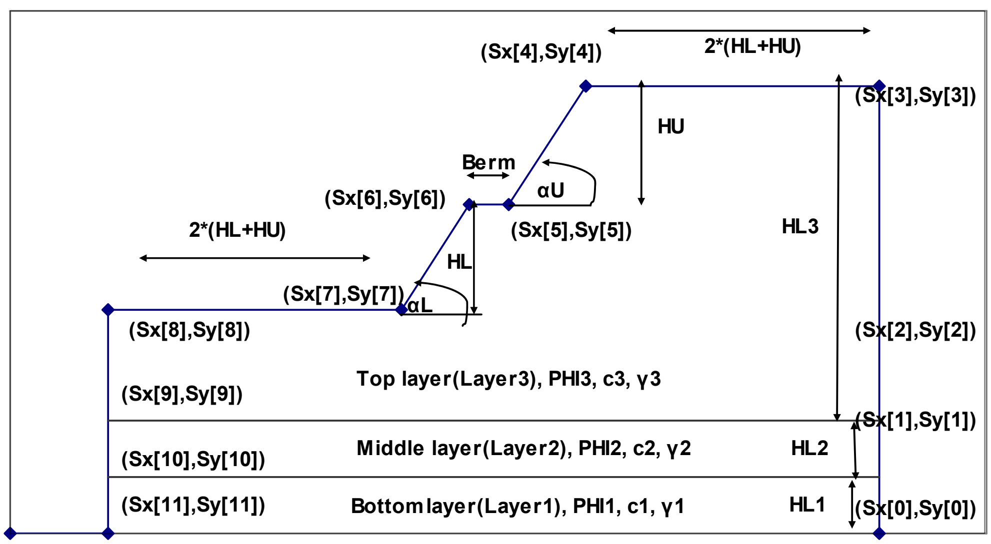

Figure 2. Cross section of slope with presence of berm.

5 Details of computer program

The program consists of three main functions and five sub functions as explained below. The three main functions which are used to evaluate three types of slope failures based on the location of slip surface as Base failure, Toe failure and Slope failure.

The above three main functions are combined into main program which calls each of the main function and gets the factor of safety. Three main functions returns three values of factor of safety in three types of failures and the programs selects the least among them. The main function has five sub functions and the main function performs the function of organizing the following set of five sub functions to get the final solutions, i.e., the calculation of factor of safety of critical slip surface. The set of five sub functions perform specific work and they are:

1 UNCON: For implementing the Davidon – Fletcher – Powell method of unconstrained minimization.

2 ONEDIM: For implementing the cubic interpolation method of one dimensional search.

3 GRADT: For evaluating the gradient of function using forward finite differences method.

4 FTN: For providing the values of Φ, f and G_j, j = 0, 1, 2… M-1 corresponding to any design vector X. It also performs automatic calculation of input data and expressing objective function and constraints in terms of design variables (X [0], X [1], X [2]……..X [N-1]).

5 PRESSURE: For the given slope cross section it calculates the total earth pressure at the bottom of each slice and also the porewater pressures of each slice. The function returns the pressure values in an array called PR [ ] and pore water pressure in U [ ].

Another program is developed which takes care of tension crack in the top layer of slope. Nowadays the height of slope is very high. In such cases the slopes are accommodated with berm in between. To study the effect of berm in stability of stratified soil slopes, a separate program is developed. In this case the effect of tension crack is also considered separately. Hence there are four types of slopes considering presence or absence of tension crack and berm. They are:

1. Slope with absence of berm and absence of tension crack – ABAT.
2. Slope with absence of berm and presence of tension crack – ABPT.
3. Slope with presence of berm and absence of tension crack – PBAT.
4. Slope with presence of berm and presence of tension crack – PBPT.

6 Validation of the Program

The output obtained by the programs is validated with the factor of safety of slopes as published in the literature. The input parameters are maintained as same as in the particular method but the type of failure surface and number of soil layers are varying in different methods. In the programs same input parameters are given for all the layers for the validation process. The factors of safety are listed in Table 1.

Table 1. List of methods with the corresponding Factor of safety.

Sl. No.	Method of Evaluation	Factor of Safety
1.	Low's method for circular slip surface (Low 1989).	1.38
2.	STABR program developed by Duncan and Wang by Simple method of slices (Philip et al 2001).	1.36
3.	Sequential unconstrained minimization technique in conjunction with Powell method (Base failure without tension crack) (Sabhahit et al 2001).	1.44
4.	Sequential unconstrained minimization technique in conjunction with Davidon Fletcher Powell method (Base failure without tension crack) (Present study).	1.34
5.	Sequential unconstrained minimization technique in conjunction with Davidon Fletcher Powell method (Base failure with tension crack) (Present study) .	1.24

The values of factor of safety in the last two rows are the one obtained using the program explained in this paper. In this program the surcharge load due to the nearby residential houses are taken as acting over the failure wedge and the pore pressure ratio. The tension crack is assumed to be filled with water is also quite effective in reducing the factor of safety. From the literature we can find for Φ more than 3˚ and the slope angle less than 53˚, the critical slip surface is invariably through toe (Karl Terzaghi et al, 1985; Venkataramaiah, 1993). In the parametric study the Φ value is more than 3˚ and the type of failure is toe failure up to 45˚ slope angle. Table 1 shows that the factor of safety from this method is comparable with the values as in published literature.

Plaxis 8.0 is a finite element program for geotechnical applications in which soil models are used to simulate the soil behavior. The software has Mohr – Coulomb model which can be applied to calculate safety factors using a 'Phi – c reduction procedure. The failure surface obtained using the program is compared with the model available in software model. The failure surface obtained using the Plaxis and the program is shown in Figure 3. Both the surfaces are almost matching with small deviations which may be due to difference in method of analysis. The factor of safety using Plaxis is 1.34 and by the developed program is 1.259 which is about 6.0% less with respect to the Plaxis value. The increase in cross sectional area of failure wedge in the developed program is 15% with respect to the Plaxis program.

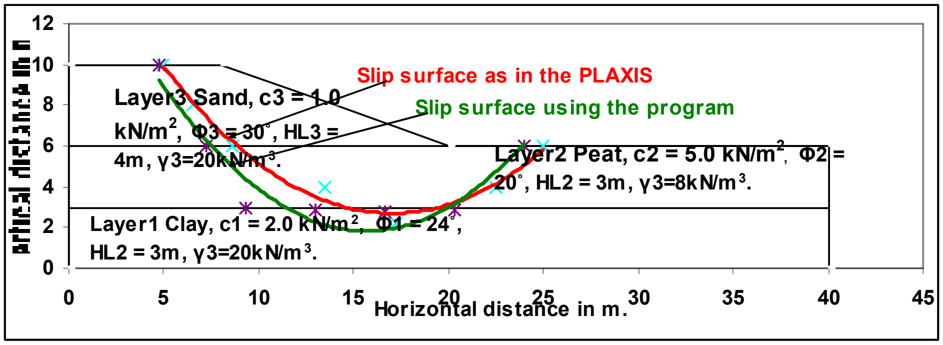

Figure 3. Slip surfaces obtained using PLAXIS and the developed program in Base failure without tension crack.

7 Results and discussion of Parametric study

A parametric study is carried out for the slope cross section as listed above by varying the input parameters one at a time from small values to higher values in steps. The parameters considered for the study are Unit weight of soil in layers, Pore water pressure ratio in soil, Height of slope, Slope angle, Shear strength parameters and Width of berm in slope with berm. The effect of variation of individual parameter is studied by keeping other parameters at constant value or constant ratio. The study is carried out for all four types but the graphs corresponding to slope with presence of berm and presence of tension crack is only presented here.
In the parametric study the effect of a particular parameter is studied by keeping the other parameters at a particular constant values. The values varied and thier range are shown in the legend and the other values are indicated in Table 2.

Table 2. Values of varoious parameters that are used in the parametric study.

Sl. No.	Parameter	Values
1.	Slope ange	AU = 45°, AL = 45°
2.	Height of slope	HU = 8.0m, HL = 7.0m
3.	Thickness of bottom, middle and top layers	HL1 = 0.25*(HU+HL), HL2 = 0.5*(HU+HL), HL3 = 1.25*(HU+HL)
4.	Friction angle of bottom, middle and top layers	Φ1 = 12°, Φ2 = 24°, Φ3 = 36°
5.	Unit cohesion of bottom, middle and top layers	c1 = 20 kN/m^2, c2 = 15 kN/m^2, c3 = 10 kN/m^2
6.	Unit weight of bottom, middle and top layers	γ1 = 21 kN/m^3, γ2 = 0.95* γ1, γ3 = 0.90* γ1
7.	Berm width	B = 4.0m
8.	Pore water pressure ratio (r_u)	0.20

7.1 Effect of unit weight of soil

Figure 4 shows the variation of factor of safety for the increase in unit weight of soil in three layers. The value of factor of safety decreases with the increase in unit weight of soil in different layers. The layered slope with berm and if the effect of tension crack is considered the stability of slope is not affected at any value of unit weight. The factor of safety reduces by 33% (2.237 to 1.558) in Base failure, 13% (1.919 to 1.679) in Toe failure and 21% (2.274 to 1.788) in Slope failure when the unit weight of bottom layer is changed from 15kN/m^3 to 24kN/m^3.

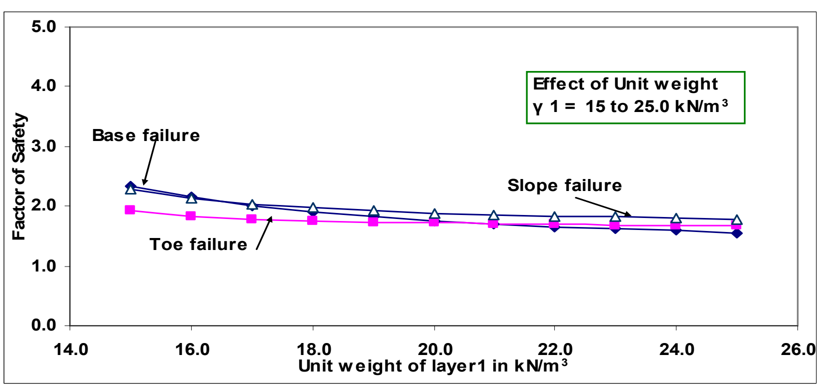

Figure 4. Effect of unit weight of soil on Factor of safety in PBPT.

8 Effect of pore pressure ratio of soil

Figure 5 shows the variation in factor of safety due to the increase in pore pressure ratio in slope with berm and with tension crack. The factor of safety is decreasing with the increase in pore pressure ratio in all the three cases of failure. The slope is unstable when pore pressure ratio reaches 0.45 and the failure type is toe failure. The factor of safety is reduced by 39% in Base failure, 53% in Toe failure and by 56% in Toe failure for the change in pore pressure ratio from 0.0 to 0.6.

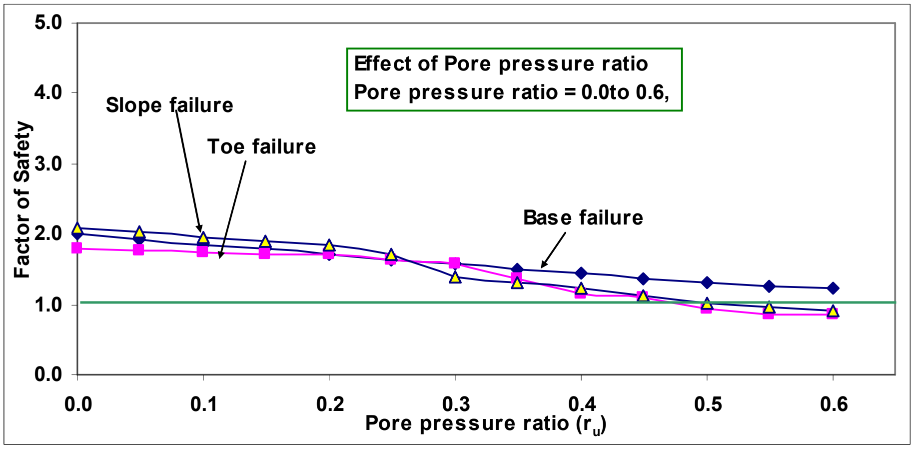

Figure 5. Effect of pore water pressure ratio of soil on Factor of safety in PBPT.

8.1 Effect of height of slope

Figure 6 shows the variation in factor of safety due to the increase in lower height of slope in slope with berm and without tension crack. The curve of factor of safety in base failure and slope failure is decreasing with the rise in lower height of slope and are almost equal. But in the case of toe failure the factor of safety increases initially and when the height reaches 5.0m the factor of safety is very high. The curve after wards decreases continuously. The factor of safety in Base failure is decreased by 56%, in Toe failure the reduction is about 30% and in Slope failure reduces by 60% when the lower height of slope is changed from 3.0m to 9.0m.

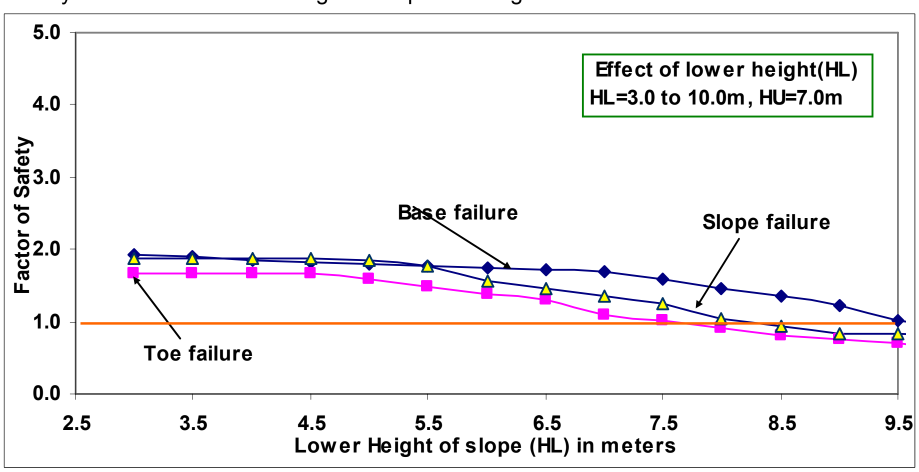

Figure 6. Effect of lower height of slope on Factor of safety in PBPT.

8.2 Effect of slope angle

Figure 7 shows the variation in factor of safety due to the increase in upper slope angle in slope with berm and with tension crack. The curve of factor of safety in all cases decreases with increase in upper slope angle. Figure 8 indicates that when the slope angle reaches 58˚ the slope becomes critical. Hence angle of slope above 58˚ is unsafe in the present context of slope and the type of failure is slope failure. The factor of safety in Base failure is decreased by 56%, in Toe failure the reduction is about 59% and in Slope failure reduces by 53% when the upper slope angle is changed from 25˚ to 75˚.

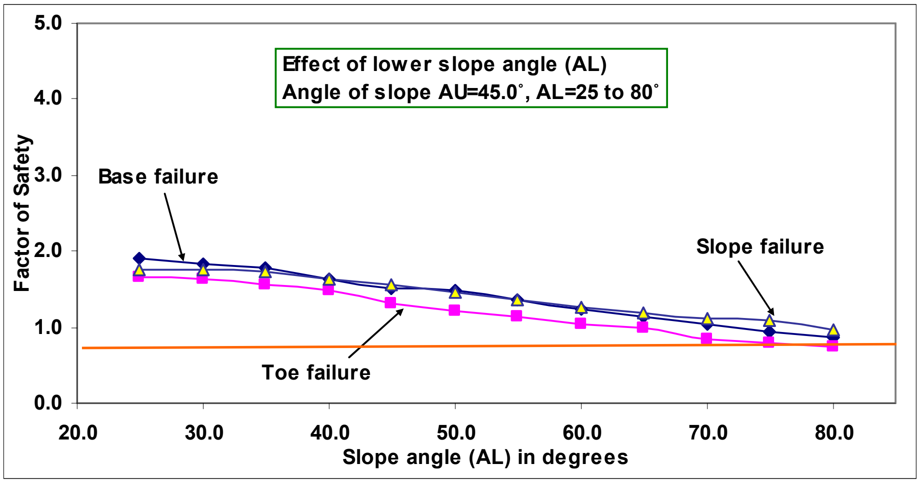

Figure 7. Effect of lower angle of slope on Factor of safety in PBPT.

8.3 **Effect of Berm**

The Figure 8 shows the variation of berm width in the slope with tension crack. In all the cases of failure the factor of safety is same at around 4.5m of berm width. From Figure 8 the width of berm about 4.5m is most effective in terms of stability. The factor of safety in Base failure is increased by 2.16 times, in Toe failure the improvement is about 1.75 times and in Slope failure improved by 2.18 times when the width of berm is increased from 0.0m to 4.0m. The factor of safety in Base failure is increased by 1.03 times (1.706 to 1.765), in Toe failure the improvement is about 1.23 times (1.403 to 1.722) and in Slope failure improved by 0.99 times (1.758 to 1.753) when the width of berm is increased from 4.0m to 10.0m.

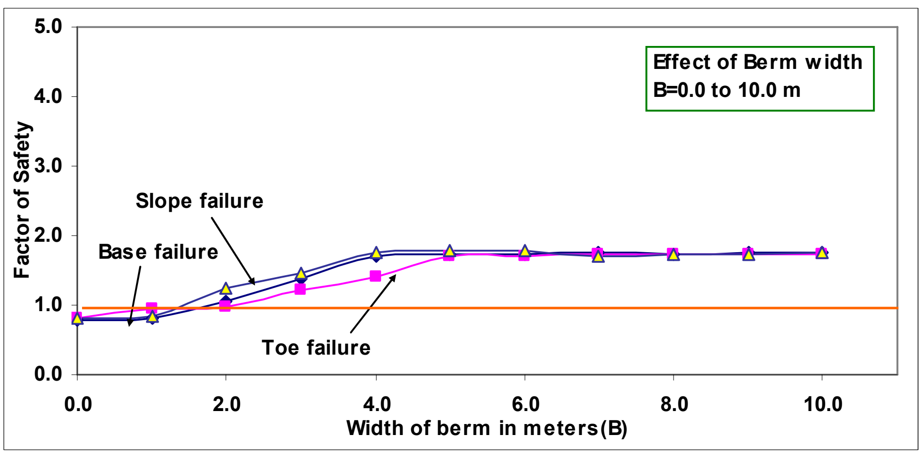

Figure 8. Effect of berm width of slope on Factor of safety in PBPT.

8.4 **Effect of friction angle**

Figure 9 shows that the factor of safety increases with the rise in friction angle of bottom layer. The increase in factor of safety in toe failure is 1.32, 1.18, 1.197, 1.34 and 1.18 times the initial value when the angle of internal friction of bottom layer (PHI1) is raised from 10˚ to 20˚ at friction angle ratio respectively at 1.0, 2.0, 3.0, 4.0 and 5.0. The factor of safety also increases with the increase in friction angle ratio. In toe failure the factor of safety increases with Friction angle ratio (FAR) by 9%.

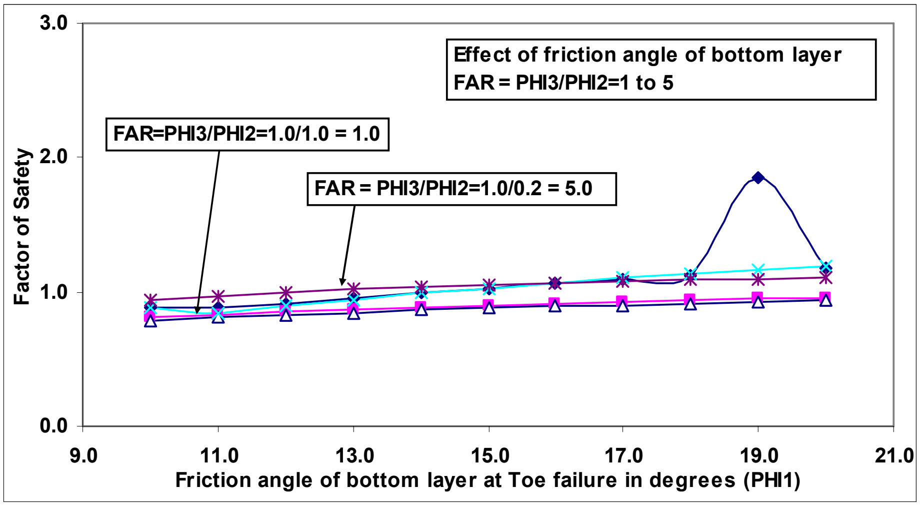

Figure 9. Effect of friction angle of bottom layer on Factor of safety in PBPT.

8.5 Effect of unit cohesion

Figure 10 shows that the factor of safety increases with the rise in unit cohesion of bottom layer. The change in factor of safety in toe failure is 1.03, 1.04, 1.05, 1.05 and 1.08 times the initial value when the unit cohesion of bottom layer (c1) is raised from 15 to 25kN/m^2 at cohesion ratio respectively at 1.0, 2.0, 3.0, 4.0 and 5.0. The factor of safety increases with very small value of about 4% with the rise in cohesion ratio.

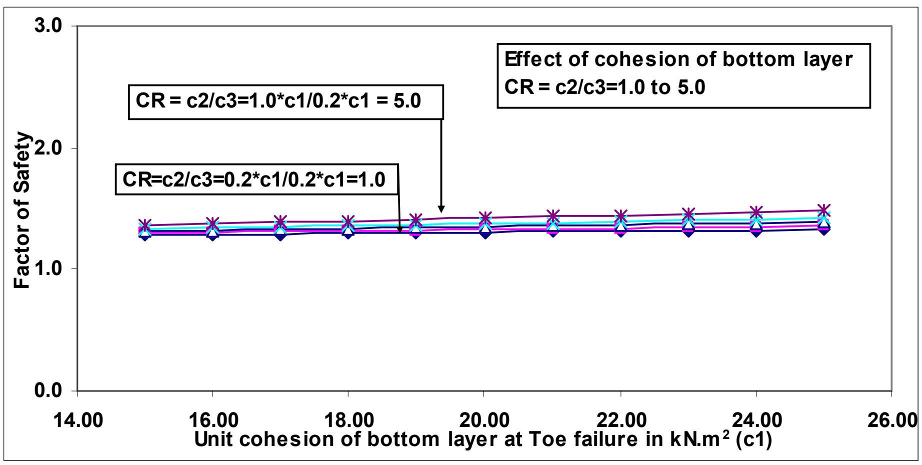

Figure 10. Effect of unit cohesion of bottom layer on Factor of safety in PBPT.

9 Conclusive Remarks:

The study is carried out to understand the stability of slopes of embankment and hills composed of stratified soils by developing computer program and the following observations and conclusions are arrived.

The developed C language computer program, which couples Janbu's procedure with the sequential unconstrained minimization technique of non-linear programming, is an efficient and effective tool for the determination of factor of safety of layered soil slopes. In addition to the factor of safety, the shape and location of the critical failure surface also come out as a part of the solution. The procedure does not involve any preassigned geometry of the shear surface and has the potential to be used for heterogeneous soil conditions under arbitrary surcharge and earthquake type of loading. The program has considered the effect of tension crack which exists in cohesive soils.

10 References

Janbu N., 1973, Slope stability computations, Embankment Dam Engineering., Casagrande Volume, John Wiley & Sons, Inc., 47 – 86.

Janbu, N., Bjerrum, L., and Kjaernsli, B., 1956, Veiledning ved Losning av Fundamenteringsoppgaver, Chapters 1 and 2, Norwegian Geotechnical Institute Publication, No. 16, Oslo.

Low, B. K., and Wilson, H. T., 1997, Probabilistic Slope Analysis using Janbu's generalized Procedure of Slices, *Computers and Geotechnics*, Vol. **21**, Issue 2, 121 – 142.

Morgenstern, N. R., and Price, V. E., 1965, The Analysis of the Stability of Generalized Slip Surfaces, *Geotechnique*, Paris, **15**, No. 1, 79 – 93.

Philip, S. K. O., and Lum, W. B., 2001,A Graphical Technique for Analysis of Slopes in C-Ø Soils, *Indian Geotechnical Journal*, **31,** (2), 198– 214.

Rao, S. S., 1991, Optimization: Theory and Applications, Wiley Eastern Limited, Second Edition, 339 – 426.

Sabhahith, N., Madhav, M. R. and Basudhar, P. K., 2001,Stability of Embankments on Non homogeneous Clay of Finite depth, *Indian Geotechnical Journal*, **31** (3), 221 – 239.

Spencer, E., 1967,A Mthod of Analysis of the Stability of Embankments Assuming Parallel Interslice Forces, *Geotechnique*, **17**, 11– 26.

Spencer, E., 1973,The Thrust Line Criterion in Embankment Stability Analysis, Geotechnique, Vol 23, 85 – 101.

Terzhaghi, K., and Peck, R. B., (1967),Soil Mechanics in Engineering Practice", 2nd Edition, John Wiley & Sons, Inc., New York, 86 – 101.

Venkataramaiah, C., 1993, "Geotechnical Engineering", 2nd Edition, New Age International (P) Limited, Publishers, New Delhi, 370 – 409.