TABLE I
SENSITIVITY VALUES FOR A TRANSMISSION LINE

Normalised sensitivity	Conductor	Parameter				
		Spacing (D)	Height (H)	Radius (R)	Frequency (f)	Length (l)
$\hat{S}_x^{V_\ell}$	I	0.0248	0.0027	-0.0249	-0.0538	-0.0317
	II	0.0162	0.0013	-0.0198	0.0402	-0.0731
	III	0.0092	-0.0003	0.0165	-0.0652	-0.0023
$\hat{S}_x^{P_\ell}$		0.0079	0.0052	-0.0126	-0.2227	-0.0534
$\hat{S}_x^{Q_\ell}$		-2.9141	-0.1046	2.9678	-13.4619	-9.0371

III. APPLICATION TO A SAMPLE SYSTEM

In order to demonstrate the utility of the theory, a 220 KV horizontal line with sending-end active and reactive powers of 150 MW and 50 MVAR with line length 300 Km is considered. The spacing D, height H, radius R, and resistance R_1 of the conductor are assumed to be 6.096 m, 7.62 m, 1.27 cm and 0.93×10^{-4} Ω/m, respectively. Table I summarises the results obtained at 60 Hz.

The reactive power at the receiving end decreases due to increase of spacing because of the increase of line reactance and decrease of the shunt susceptance. However, the voltage magnitude and therefore the regulation increases with increase of spacing. The height of the conductor and its radius influence the reactive power only. Summarising, both receiving-end voltage and power are most sensitive to length and frequency and least to height. Of all the quantities, the reactive power is affected the most with the change in the values of the parameters.

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State-Space Models Using Modified Cauer Continued Fraction

R. PARTHASARATHY AND SARASU JOHN

***Abstract*—This letter presents a procedure for obtaining reduced-order models in state-space formulation, using modified Cauer continued fraction as the basis.**

The proposed procedure is amenable to programming on a digital computer.

I. INTRODUCTION

In the past decade, the problem of deriving reduced order models for high-order plants has received considerable attention of research workers [6]. In [1], Shieh and Goldman presented a procedure for obtaining a minimal realization employing the first, second, and the mixed Cauer forms. In this letter, we employ instead the following modified Cauer form [2]:

$$g(s) = \cfrac{1}{h_1 + \cfrac{s}{k_1 + \cfrac{1}{h_2 + \cfrac{s}{\ddots}}}}$$

where $h_1, k_1, h_2, \cdots$ are scalar quotients. The minimal realization thus obtained is then used to derive the reduced-order models. The relation connecting the state variables of the original system with those of the model is also developed, which makes it possible the use of such models for deriving suboptimal control policies [6].

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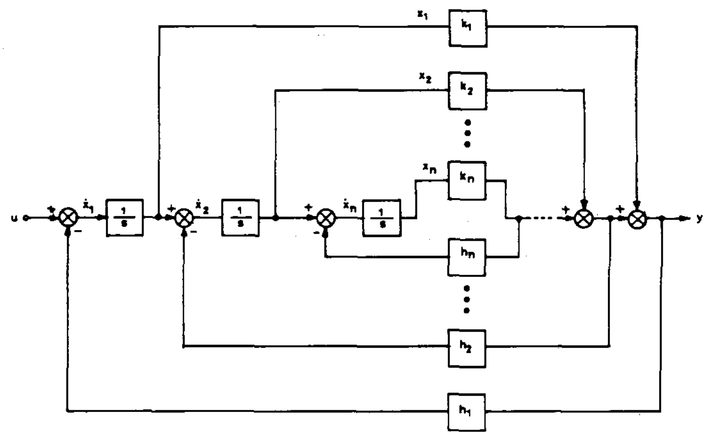

Fig. 1. Block diagram representation of an nth-order system.

II. MODEL REDUCTION PROCEDURE

Consider a linear time-invariant system represented by

$$g(s) = \frac{y(s)}{u(s)} = \frac{b_{11} + b_{12}s + \cdots + b_{1,n}s^{n-1}}{a_{11} + a_{12}s + \cdots + a_{1,n}s^{n-1} + s^n} \tag{1}$$

$g(s)$ is then expanded in the modified Cauer form by constructing the following table:

MODIFIED ROUTH TABLE

$$\begin{array}{lllll}
 & a_{11} & a_{12} \cdots & a_{1,n-1} & a_{1,n} \; 1 \\
h_1 = \dfrac{a_{11}}{b_{11}} & & & & \\
 & b_{11} & b_{12} \cdots & b_{1,n-1} & b_{1,n} \quad k_1 = b_{1,n} \\
 & a_{21} & a_{22} \cdots & a_{2,n-1} & 1 \\
 & \vdots & \vdots & \vdots & \vdots \\
 & a_{n,1} & 1 & & \\
h_n = \dfrac{a_{n,1}}{b_{n,1}} & & & & \\
 & b_{n,1} & \quad k_n = b_{n,1}. & & \\
 & 1 & & &
\end{array}$$

The denominator and numerator coefficients of (1) form the first two rows. The successive rows are evaluated using the relations [3]:

$$a_{j+1,k} = a_{j,k+1} - h_j b_{j,k+1}$$

and

$$b_{j+1,k} = b_{j,k} - k_j a_{j+1,k}, \quad j = 1, 2, \cdots, n-1; k = 1, 2, \cdots, n-j \tag{2}$$

where

$$h_j = \frac{a_{j,1}}{b_{j,1}}$$

and

$$k_j = \frac{b_{j,n+1-j}}{a_{j+1,n+1-j}}, \quad j = 1, 2. \cdots, n.$$

A block diagram interpretation of this form of continued fraction is given in Fig. 1. From the figure a realization for (1) can be directly written as

$$\dot{v} = Hv + Lu \quad y = D_p v \tag{3}$$

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where

$$H = -\left[\begin{array}{cccc:ccc} h_1k_1 & h_1k_2 & \cdots & h_1k_r & \cdots & h_1k_{n-1} & h_1k_n \\ -1 & h_2k_2 & \cdots & h_2k_r & \cdots & h_2k_{n-1} & h_2k_n \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & h_rk_r & \cdots & h_rk_{n-1} & h_rk_n \\ \hdashline \vdots & \vdots & & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & -1 & h_nk_n \end{array}\right]$$

$$L = \left[\begin{array}{c} 1 \\ 0 \\ \vdots \\ 0 \\ \hdashline \vdots \\ 0 \end{array}\right]$$

and

$$D_p = [k_1 \quad k_2 \quad \cdots \quad k_r \,\vdots\, \cdots \quad k_{n-1} \quad k_n].$$

A phase canonical form of realization for (1) is

$$\dot{x} = Ax + Bu, \quad y = Dx \tag{4}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -a_{11} & -a_{12} & -a_{13} & \cdots & -a_{1,n} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

and

$$D = [b_{11} \quad b_{12} \quad b_{13} \quad \cdots \quad b_{1,n}].$$

It is found that a transformation matrix P, defined as $v = Px$, transforms the triple (A, B, D) to (H, L, D_p) where the matrix P is obtained as

$$P = \begin{bmatrix} a_{21} & a_{22} & \cdots & a_{2,n-1} & 1 \\ a_{31} & a_{32} & \cdots & 1 & \\ \vdots & \vdots & & & \\ a_{n,1} & 1 & & & \\ 1 & & & & \end{bmatrix}. \tag{5}$$

The rows of P are copied from the 3rd, 5th, $\cdots$, $(2n + 1)$th rows of the modified Routh Table. It is evident that

$$H = PAP^{-1} \qquad L = PB \text{ and } D_p = DP^{-1}. \tag{6}$$

Following the modelling technique given in [3], a reduced-order model of dimension r can be obtained by truncating the continued fraction after $2r$ quotients. In the block diagram, this is equivalent to discarding all the inner loops beyond the rth loop. Hence the state-space description of the model can be written as

$$\dot{z} = Fz + Gu \qquad y = D_m z \text{ and } z = Tv \tag{7}$$

where the matrices (F, G, D_m) are submatrices of (H, L, D_p) as partitioned in (3). Thus the truncation of the continued fraction results in the following relations:

$$F = THT^+ \quad G = TL \quad D_m = D_pT^+ \tag{8}$$

where $T = [I_r \,\vdots\, 0]$ and $TT^+ = I_r$, I_r denoting an identity matrix and

$$z = (TP)x. \tag{9}$$

Equation (9) connects the state variables of the original system and the reduced-order model.

III. Conclusion

A simple procedure for obtaining reduced-order models directly in the state-space form via the modified Cauer continued fraction is presented. It may sometimes produce unstable models, even if the original system is asymptotically stable. The models match a set of initial time-moments and Markov parameters of the original system [4], which ensures that the outputs of the system and the model follow closely both the transient and steady state portions. It is found that the realization obtained here is identical to that in [5].

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A Simple Sufficient Criterion for the Stability of Multidimensional Digital Filters

H. C. REDDY, P. K. RAJAN, AND M.N.S. SWAMY

***Abstract*—In this paper a theorem concerning the absence of zeros of a multivariable polynomial in the closed unit polydisc of a multidimensional complex plane is presented. Using this theorem the stability of a large class of multidimensional digital filters can be tested almost by inspection.**

Introduction

There is a growing need for the development of simple stability testing procedures for multidimensional (n-D) digital filters. There are a number of general testing procedures available in the literature [1]-[4]. In this letter a simple sufficiency test is given for testing the stability of a n-D digital filter of any order. This is achieved by converting an $(m + 1)$ term denominator polynomial of the filter transfer function into an m-variable first-degree polynomial.

Notation:
$$\overline{U}^k = \{z_1, \cdots, z_k\}: |z_1| \leqslant 1, \cdots, |z_k| \leqslant 1\},$$
$$\overline{T}^k = \{z_1, \cdots, z_k\}: |z_1| = 1, \cdots, |z_k| = 1\},$$
$$R_j = \{0, 1, 2, \cdots, N_j\}$$

Theorems on Testing: We will first consider a theorem which gives the necessary and sufficient condition for the absence of zeros in $\overline{U}^k$ for an m-variable first-degree polynomial of a special form.

Theorem 1: An m-variable digital polynomial

$$B(z_1, \cdots, z_m) = \sum_{i=1}^{m} b_i z_i + b_0 \neq 0 \text{ in } \overline{U}^m$$

$$\text{iff } |b_0| > \sum_{i=1}^{m} |b_i|.$$

Proof: Without loss of generality we assume $b_0 > 0$. Thus $|b_0| = b_0$. The necessity can be proved from the following:

Let

$$z_i = \gamma_i e^{j\theta_i} B(z_1, \cdots, z_m)$$

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