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# Enhancing the Error-correcting Capability of Imai-Kamiyanagi Codes for Data Storage Systems by Adopting Iterative Decoding using a Parity Check Tree

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## ABSTRACT

A novel low-complexity, soft decision technique which allows the decoding of distance-5 double error-correcting Imai-Kamiyanagi codes by using a parity check tree associated with the Tanner graph is proposed. These codes have been applied to memory subsystems and digital storage devices in order to achieve efficient and reliable data processing and storage. For the AWGN channel, gains in excess of 1.5 dB at reasonable bit error rates with respect to conventional hard decision decoding are demonstrated for the (46, 32), (81, 64), and (148, 128) shortened Imai-Kamiyanagi codes.

### Keywords:

*Imai-Kamiyanagi codes, Iterative decoding, Memory architecture, Parity check tree, Tanner graph.*

## 1. INTRODUCTION

Channel coding is commonly used to achieve reliable transmission of information. Error control codes enable a decoder to recover from errors produced by noise in a communication channel/storage device. Codes ensure higher noise tolerance at the receiver by adding redundancy into the user data to achieve better separation of data sequences. Error control coding (ECC) algorithms have contributed enormously to the communication revolution. It is the development of ECC algorithms and techniques have made the internet and digital recording ubiquitous facilities which can be enjoyed by all people. In addition, they have also contributed significantly to deep space exploration and the explosion in human knowledge about the various planets and satellites in the solar system. The last fifteen years have been characterized by research work which has enabled exceptional increase in data transmission rates and storage capabilities. Increased use of cellular wireless communication in an environment plagued by spectrum shortage has resulted in the development of several innovative and novel ideas to secure information integrity and enhance channel capacity. There has also been rapid development in microelectronics. Both of these developments often going hand in hand has provided us with both a need for and the possibility of implementing sophisticated algorithms for error control [1].

The advent of the internet age has produced enormous demand for increases in storage capacity and storage density. Physical/Media improvements along with sophisticated signal processing and coding techniques have played a critical role in the constant augmentation of storage/communication channel capacities. Every computer memory and data storage systems have adopted some types of error-detecting or error-correcting codes in order to enhance system reliability. The reliability levels that are required by storage devices are extremely high since unlike communication systems generally no retransmission is possible. We expect to save our data and be able to retrieve it perfectly at any future time. It is the art of error control and correction that makes this possible [2]. Permanent and temporal faults are the major sources of errors in modern digital storage systems. Power supply break down, defective open or short circuits, bridging or open lines, electron-migration, etc., cause permanent faults. Permanent fault leads to hard errors; they therefore affect the system functions for a long period of time. Temporal faults can be transient or intermittent. Transient faults occur randomly and externally because of external noise, namely electromagnetic waves and also particles such as  $\alpha$ -particles and neutrons. Intermittent faults occur randomly but internally because of unstable or marginally stable hardware, varying hardware or software state as a function of load, or signal coupling (i.e., crosstalk) between adjacent signal lines. Some

intermittent faults may be due to glitches which are unpredictable spike noise pulses occurring and propagated, especially in large combinational digital circuits. Temporal fault leads to soft errors. Soft errors have a limited duration, meaning they interrupt system functions for a very short time period. Therefore, soft errors are also called transient errors. Some reports show that more than 60% of all failures in computer systems are caused by transient or intermittent faults. For example, in dynamic random access memory (DRAM) chips, transient errors result mainly from  $\alpha$ -particles emitted by the decay of radioactive particles. As they pass through the chip,  $\alpha$ -particles create sufficient electron-hole pairs to add charge to the DRAM capacitor cells. These particles have low energy level, and thus have very low probability of causing more than one memory cell to flip when the memory cells are not packed in extreme density. In today's ultra high-density RAMs, not only DRAMs but also static RAMs, it has been recognized that multiple cosmic ray-induced transient errors are a serious problem [3-6].

Error detection is an essential part of a storage system design. Ideally, error detection will block the propagation of an error during online operations, before it reaches the system interface and causes a system failure. The error is best be detected immediately as it occurs so that its effect can be minimized. Thus, the use of error control algorithms can protect the integrity of user information against errors caused by aging, wear out due to repeated read and write operations, electromagnetic waves, and manufacturing defects. Many different error control codes have been studied and developed to correct and/or detect the types of errors mentioned above. The coding approach involves some redundancy, for example, additional check bits, additional hardware in the form of encoding/decoding logic circuits, and additional decoding time delay. Nevertheless, the coding performance is superior to those competitive techniques, especially in quickly masking of temporal faults [7,8].

## 2. CONSTRUCTION OF IMAI-KAMIYANAGI CODES

According to the manner in which redundancy is added to messages, error-correcting codes can be divided into two classes: Block and Convolutional. Block codes implement a one-to-one mapping of a set of  $K$  information symbols on to a set of  $N$  code symbols. We call this code is an  $(N, K)$  linear block code. The  $R = N - K$  symbols in a codeword are a function of the information symbols, and provide redundancy that can be used for error correction and/or detection purposes. The minimum distance  $d_{\min}$  of a block code  $C$  is the smallest Hamming distance between any two

codewords in the code. Imai and Kamiyanagi added an extended structure to the double error-correcting (DEC) BCH codes. The Imai-Kamiyanagi code has  $d_{\min} = 5$ . Hence, the hard decision decoding (HDD) of these codes correct any two bit random errors in a block of size  $N$ . Although the Imai-Kamiyanagi code has slightly larger check-bit length, the parity check matrix of the code is very sparse (i.e., a small portion of the entries being one, all others being zero). This is why the decoding hardware has less complexity and decoding can be made faster than DEC-BCH code [8]. BCH codes have a minimum or smaller number of check bits, but have complex decoding hardware and a longer decoding time. Today's high-speed computer systems have adopted the information-bit lengths  $K = 32, 64,$  or  $128$  bits. In Table 1, we have given a comparison of pertinent parameters of shortened BCH and Imai-Kamiyanagi codes for data blocks of size 32, 64, and 128 bits.

The parity check matrix  $H$  of general Imai-Kamiyanagi code is given by [8],

$$H = \begin{bmatrix} H'_1 \\ H'_2 \\ H'_3 \end{bmatrix} \tag{1}$$

where,  $H'_1, H'_2$  and  $H'_3$  are defined as

$$\begin{aligned} H'_1 &= [A \otimes H_1 \quad 0_{m \times r_1}] & H'_2 &= [B \otimes 1_n \quad I_{r_1}] \\ H'_3 &= [1_{n_0} \otimes H_3 \quad 0_{m \times r_1}] \end{aligned} \tag{2}$$

Here, symbol  $\otimes$  depicts the Kronecker product. The meaning of other notations is defined here:

- $1_n$  : all-1  $n$ -tuple row vector, and  $n = 2^m - 1, m \geq 2$
- $1_{n_0}$  : all -1  $n_0$ -tuple row vector,
- $0_n$  : all-0  $n$ -tuple row vector,
- $I_{r_1}$  :  $r_1 \times r_1$  identity matrix,
- $0_{m \times r_1}$  :  $m \times r_1$  zero matrix,
- $A$ :  $r_0 \times n_0$  matrix defined over Galois Field  $F_2$ . The rightmost  $r_0$  column vectors are linearly independent.
- $B$ :  $r_1 \times n_0$  matrix over  $F_2$ . This has  $r_1$  linearly independent column vectors.

**Table 1: Comparison of shortened BCH and Imai-Kamiyanagi code parameters**

Codes	Code length, $N$	Data length, $K$	Check bits, $N - K$	$d_{\min}$
BCH (Shortened)	44	32	12	}5
	78	64	14	
	144	128	16	
Imai -Kamiyanagi (Shortened)	46	32	14	}5
	81	64	17	
	148	128	20	



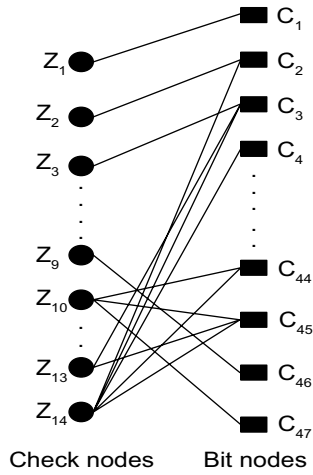


Figure 1: Tanner graph of a (47, 33) Imai-Kamiyanagi code.

The leitmotiv here is soft decoding of Imai-Kamiyanagi codes described through graphical structures. A graph such as this, consisting of two distinct sets of nodes and having edges only between the nodes in different sets, is called a bipartite graph or Tanner graph. The Tanner graph is used to develop insight into the decoding algorithm. The iterative soft decoding algorithm to decode Imai-Kamiyanagi codes using a parity check tree associated with the Tanner graph is explained below.

For each code bit  $c_n$ , compute the checks for those checks that are influenced by  $c_n$ . To do this, we propagate probabilities through the Tanner graph, thereby accumulating the evidence that the checks provide about the bits. Suppose that  $c_n$  is in error and that other bits influencing its checks are also in error. Let arrange the Tanner graph with  $c_n$  as a root shown in Figure 2. This rearrangement of Tanner graph is called a parity check tree. The situation is particularly simple if the graph is a tree, i.e., it is connected, but if any one edge is removed, the graph is no longer connected, in particular a tree has no circuits (A closed path containing at least one edge is called a circuit). Thus, for each edge, we can talk about the sub graph on either side of it. Assume the bits in the star mark are in error. The bits that connect to the checks connected to the root node are said to be in level 1. The bits that connect to the checks from the first level are said to be in level 2. We can establish many such levels. Then, decode by proceeding from the leaves of the tree (right most part of the figure). By the time decoding on  $c_n$  is reached, other erroneous bits may have been corrected. Thus, bits and checks which are not directly connected to  $c_n$  still influence  $c_n$  [12].

#### 4. MATHEMATICAL DESCRIPTION OF DECODING ALGORITHM

In this section, mathematical description of iterative decoding algorithm for DEC Imai-Kamiyanagi codes

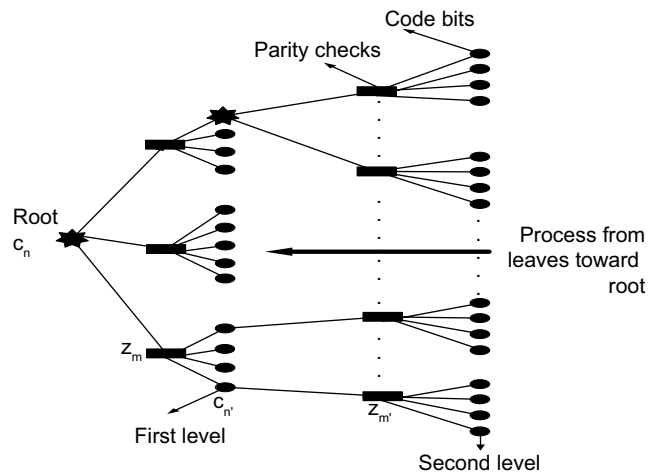


Figure 2: Two-level parity check tree associated with the Tanner graph.

by traversing through parity check tree is presented. The following notation is convenient in describing the algorithm [13-16]. Let  $h_{i,j}$  denote the entry of H in the  $i^{th}$  row and  $j^{th}$  column. Then,

$$L(m) = \{l : h_{m,l} = 1\} \tag{4}$$

denote the set of code positions that participate in the  $m^{th}$  parity check equation, and let

$$M(l) = \{m : h_{m,l} = 1\} \tag{5}$$

denote the set of check positions in which code position  $l$  participates. The algorithm iteratively computes two types of conditional probabilities:

$q_{m,l}^x$ , the probability that the  $l^{th}$  bit of codeword  $\mathbf{c}$  has the value  $x$ , given the information obtained via the check nodes other than check node  $m$ .

$r_{m,l}^x$ , the probability that a check node  $m$  is satisfied when bit  $l$  is fixed to a value  $x$  and the other bits are independent with probabilities  $q_{m'l}^x, l' \in L(m) \setminus l$ .

In the following, BPSK transmission over AWGN channel is assumed. Modulated symbols  $m(c_i) = (-1)^{c_i} \sqrt{E_s}$  are transmitted over an AWGN channel and received as  $r_i = m(c_i) + w_i$ , where  $w_i$  is a Gaussian distributed random variable with zero mean and variance  $N_0/2, 1 \leq i \leq N$ .

Initializaion:

For  $l \in \{1, 2, 3, \dots, N\}$ , initialize the apriori probabilities of the code nodes

$$p_l^1 = \frac{1}{1 + \exp(r_l \frac{4}{N_0})} \tag{6}$$



and  $p_l^0 = 1 - p_l^1$ . For every  $(l, m)$  such that  $h_{m,l} = 1$ ,

$$q_{m,l}^0 = p_l^0; q_{m,l}^1 = p_l^1. \tag{7}$$

Horizontal Step: Updating  $r_{ml}^x$

For each  $l, m$  compute

$$\delta r_{m,l} = \prod_{l' \in L(m) \setminus l} (q_{m,l'}^0 - q_{m,l'}^1), \tag{8}$$

and

$$r_{m,l}^0 = (1 + \delta r_{m,l}) / 2; r_{m,l}^1 = (1 - \delta r_{m,l}) / 2 \tag{9}$$

Vertical Step: Updating  $q_{ml}^x$

For each  $l, m$  compute

$$q_{m,l}^0 = p_l^0 \prod_{m' \in M(l) \setminus m} r_{m',l}^0; q_{m,l}^1 = p_l^1 \prod_{m' \in M(l) \setminus m} r_{m',l}^1 \tag{10}$$

and normalize, with  $\alpha = \frac{1}{(q_{m,l}^0 + q_{m,l}^1)}$ ,

$$q_{m,l}^0 = \alpha q_{m,l}^0; q_{m,l}^1 = \alpha q_{m,l}^1. \tag{11}$$

For each  $l$ , compute the a posteriori probabilities

$$q_l^0 = p_l^0 \prod_{m \in M(l)} r_{m,l}^0; q_l^1 = p_l^1 \prod_{m \in M(l)} r_{m,l}^1, \tag{12}$$

and normalize, with  $\alpha = \frac{1}{(q_l^0 + q_l^1)}$ ,

$$q_l^0 = \alpha q_l^0; q_l^1 = \alpha q_l^1. \tag{13}$$

Final Step:

Make a tentative decision: Set  $\hat{c}_n = 1$ , if  $q_l^0 > 0.5$ , else set  $\hat{c}_n = 0$ . If  $H\hat{c} = 0$ , Stop.

The majority of the errors in the byte-organized

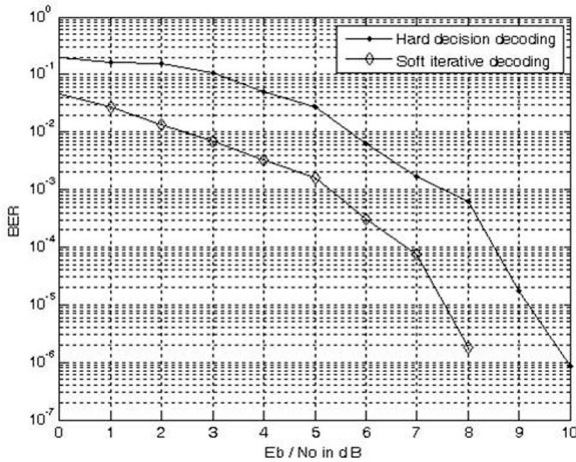


Figure 3: Performance for the (46, 32) Imai-Kamiyanagi code over AWGN with HDD and SDD.

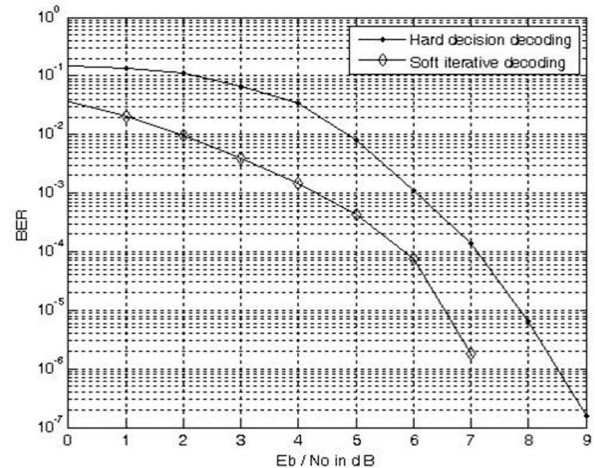


Figure 4: Performance for the (81, 64) Imai-Kamiyanagi code over AWGN with HDD and SDD.

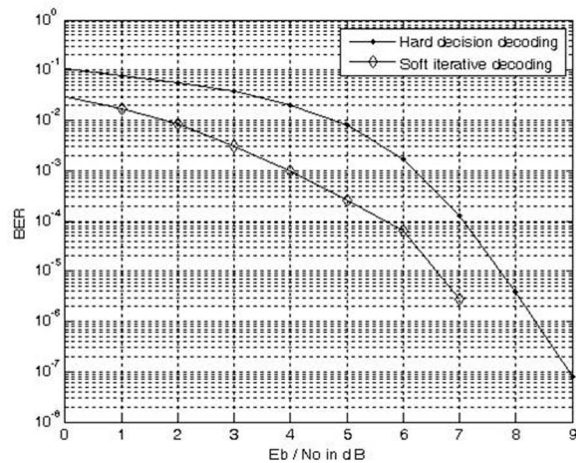


Figure 5: Performance for the (148, 128) Imai-Kamiyanagi code over AWGN with HDD and SDD.

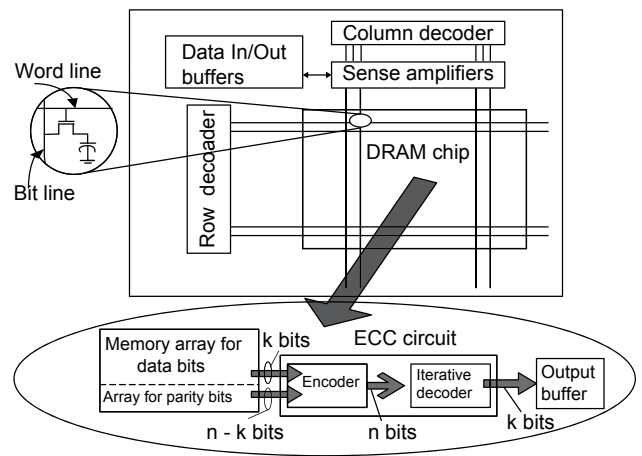


Figure 6: DRAM cell architecture with Imai-Kamiyanagi coder-decoder.

semiconductor memory systems are independent random bit errors, which may be caused by  $\alpha$ -particles, cell failures, or external noises, which is equivalent to the AWGN channel model. We have simulated (46, 32), (81, 64), and (148, 128)-shortened Imai-Kamiyanagi codes under AWGN channel conditions. Soft iterative decoding using a parity check tree is compared with contemporary HDD. Results [Figures 3-5] show almost 1.5 to 2.0 dB improvement at a bit error rate of  $10^{-6}$ . Code exhibits 3 dB gain in lower SNR region. This gain is very significant and tells that although the error correcting capability of the code is 2 bit, the code was able to decode beyond the minimum distance and correct more than two bit errors.

The latest semiconductor DRAM chips have a multi-bank architecture. Each bank is constructed with highly independent subdivided memory arrays called memory subarrays [17]. The entire output of DRAM chip is called a block and block size may be 32, 64, or 128 bits. Figure 6 shows the architecture of DRAM chip with proposed soft iterative Imai-Kamiyanagi coder-decoder.

## 5. CONCLUSION

A moderately simple and efficient soft decision-decoding scheme for Imai-Kamiyanagi code which employs parity check tree representation has been proposed. It is noteworthy that soft iterative decoder always detect the occurrence of decoding errors. The decoding complexity seems to be on the higher side. As technology progresses, power consumption of a decoder can be made very low. However, it is well known that the use of soft decision decoding on channels perturbed by Gaussian noise improves the performance by 1.5 to 3 dB over the use of HDD. Thus, a typical application for the soft iterative decoding algorithm of Imai-Kamiyanagi codes could be in mass memories, computer sub memory systems, and data entry systems. Multiple hard and soft error problems can be solved by employing this decoding method. In wireless applications, 3 dB coding gain over un-coded transmission means that data throughput can be increased by a factor of 2 for a fixed SNR. Equivalently, transmitter power can be reduced by a factor of 2 if increase in throughput is not desired. This translates into smaller transmit antennas or, alternatively, smaller

receive antennas for the same transmission power. Therefore, collectively we can say that coding gain increases the system performance or reduces cost or both.

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