Assessment of Distributed Generation Source Impact on Electrical Distribution System Performance

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***Abstract*—The recent trends in electrical power distribution system operation and management are aimed at improving system conditions in order to render good service to the customer. The reforms in distribution sector have given major scope for employment of distributed generation (DG) resources which will boost the system performance. This paper proposes a heuristic technique for allocation of distribution generation source in a distribution system. The allocation is determined based on overall improvement in network performance parameters like reduction in system losses, improvement in voltage stability, improvement in voltage profile. The proposed Network Performance Enhancement Index (NPEI) along with the heuristic rules facilitate determination of feasible location and corresponding capacity of DG source. A Priority list is prepared with decreasing values of NPEI so that the designer can select most feasible location. The developed approach is tested with different test systems to ascertain its effectiveness.**

***Index Terms*—Distributed Generation, Network Performance, Optimal Operation, Power Loss Reduction, Voltage Stability**

I. INTRODUCTION

The distribution system being the nearest link to the consumers, utmost importance is to be given for its satisfactory operation. Though this sector was neglected earlier, in recent years distribution sector reforms have been implemented and it is expected to improve the scenario in coming years. The reforms are aimed at improvement in network operation vide taking appropriate steps such as incorporation of generation sources at distribution system level and encouragement to Independent Power Producers. This will facilitate reduction in power losses of the system and the customers can be served more efficiently.

There are different technologies which can be adopted for DG sources like photo voltaic cells, wind generation, combustion engines, fuel cells and other types of generation from the resources which are available in the geographical area. The concept of distributed generation helps to harness the natural resources and help the distribution system to get more strength in its operation and control. The advantages of distributed generation are reduced system losses, improvement in voltage profile, better power quality and release of burden on the existing transmission and distribution network. In this manner the overall system reliability is improved by employing distribution generation sources in distribution network.

The presence of DGs in the distribution network improves the system voltage stability [1]. The static voltage stability is evaluated on the load flow computations and impact of DGs on the network can be accessed. It is necessary to devise appropriate model in order to reflect the generation technology employed. The DG location can be arrived at so that system voltage stability can be improved. The improper location and capacity of DG may lead to deteriorated performance in terms of increased power losses and poor voltage profile [2]. Hence it is necessary to ensure that the selection of DG capacity improves the operation of the system. This aspect demands the usage of optimum DG allocation techniques considering power loss minimization. The index to estimate the voltage profile improvement with the incorporation of DG sources gives an indication of choice on placement of such sources [3]. In conjunction with voltage profile improvement, the line loss reduction also serves as a parameter in the location and sizing issues [4]. The technical benefits can be quantified by evaluating a composite index which comprises of improvement in different parameters [5].

This paper presents a heuristic approach to address DG location and sizing issue. The proposed Network Performance Enhancement Index (NPEI) gives an indication of best location and capacity for DG source. The technique incorporates heuristic rules to evaluate most feasible location based on NPEI so that all network performance parameters are improved. In certain cases the best location may not be feasible for insertion of DG sources due to geographical, social constraints. Hence it is essential to work out other alternatives. To cater this need, a priority list of bus numbers is prepared with decreasing values of NPEI. This aspect gives flexibility for the designer to select the appropriate location satisfying technical as well as implementation constraints. The approach is applied to two test systems comprising of 33 bus and 90 bus respectively for testing the developed approach.

II. POWER FLOW ANALYSIS

The implicit Z-bus method is used for power flow analysis [6]. Since the distribution system with generation sources needs to be modeled with PV buses, the sensitivity matrix approach is adopted. As deduced in the sensitivity matrix technique, the sensitivity matrix M is computed from the impedance matrix Z [1].

The Sensitivity Matrix M is deduced as:

$$M\,\Delta\overline{Q} = \Delta\overline{V} \tag{1}$$

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Where $\Delta\overline{Q}$ is reactive power mismatch and $\Delta\overline{V}$ is the voltage mismatch of PV nodes. Since PV buses have voltage nearly of 1.0 p.u , the reactive power mismatch will be approximately equal to injected current increment and hence sensitivity matrix M will be :

$$M = -\begin{bmatrix} |Z_{11}| & |Z_{12}| & \cdots & |Z_{1n}| \\ |Z_{21}| & |Z_{22}| & \cdots & |Z_{2n}| \\ \vdots & \vdots & \cdots & \vdots \\ |Z_{n1}| & |Z_{n1}| & \cdots & |Z_{nn}| \end{bmatrix} \quad (2)$$

While conducting the power flow using Implicit-Z Bus technique the following steps are executed:

1. The nodal current injections for all loads are computed.
2. The bus voltages are updated after computing voltage deviation vector.
3. Power mismatch and voltage mismatch is computed at each bus.
4. If power mismatch or voltage mismatch is with-in tolerance limit then power flow is said to be convergent and system power flows and line losses are computed.
5. If the mismatch is above the tolerance limit, then voltage of all nodes is updated with values of previous iteration and reactive power mismatch vector is updated with sensitivity matrix M. The power flow analysis is again carried out till convergence is reached.

III. VOLTAGE STABILITY PERSPECTIVE

The distributed generation source allocation can be done with the purpose of improving the voltage stability of the network. The improvement in voltage stability assumes importance because of the following factors.

1. A system with lesser voltage stability may move to uncontrollable state.
2. Since the existing networks are heavily loaded, the operator needs to have clear knowledge on system behavior to protect power quality.

The static voltage stability index (VSI) helps to estimate the health of the network [1]. The value of this index ranges from zero (No-Load) to 1 (Voltage Collapse). The VSI is computed as

$$L_j = \frac{4\left[\left(XP_j - RQ_j\right)^2 + \left(XQ_j + RP_j\right)V_i^2\right]}{V_i^4} \quad (3)$$

$$\text{VSI} = \max\left\{L_1, L_2, ..., L_{n-1}\right\} \quad (4)$$

Where Pj, Qj are real and reactive power received at j^{th} bus and R and X are the resistance and reactance of the branch linking jth bus. The branch corresponding to the highest index value of Voltage Stability Index (VSI) is called the weakest branch. The voltage collapse must start from the weakest branch. If the system has lower VSI then system is considered to be more stable.

IV. DISTRIBUTED GENERATION ALLOCATION

The placement of DG helps to reduce the power loss in the network. The optimum DG allocation can be treated as optimum active power compensation, like capacitor allocation for reactive power compensation. It is observed that for a particular bus, as the size of DG is increased, the losses are reduced to a minimum value up to a certain size of DG which can be treated as optimal size of DG. If the DG capacity is increased beyond the optimal size of DG then the system losses are again increased above the minimum loss magnitude. Hence the location and sizing of DG from loss reduction perspective has to be done carefully. The optimum size of DG varies from one bus to another bus. The best location can be chosen as the bus where the optimum DG capacity injection gives the highest loss reduction. The determination of optimal DG capacity for every bus has to be done to arrive at the final decision on location and sizing of DG in the distribution network.

V. OPTIMAL CAPACITY OF DG SOURCE

This methodology based on loss formula is adopted for computation of optimal capacity of DG source [2]. This approach requires load flow to be executed two times, first time for the base case without DG and another time with the optimal capacity of DG inserted in the network at a particular bus. The technique uses loss coefficients and the optimal capacity of DG is computed as below:

Since at lowest power loss, the rate of change of rate of change of losses PL with respect to injected power Pi becomes zero.

$$\frac{\partial P_L}{\partial P_i} = 2\sum_{j=1}^{n}(\alpha_{ij}P_j - \beta_{ij}Q_j) = 0 \quad (5)$$

α_{ij} and β_{ij} are loss coefficients. 'j' varies from 1 to n branches, P_j and Q_j are real and reactive power flows in the branches.

The optimal Capacity of DG is computed as

$$P_{dgi} = P_{li} + \frac{1}{\alpha_{ii}}\left[\beta_{ii}Q_i - \sum_{j=1, j\neq i}^{n}(\alpha_{ij}P_j - \beta_{ij}Q_j)\right] \quad (6)$$

Where P_{dgi} is the real power injection from DG placed node i, and P_{li} is the Load at node i.

The algorithm for optimum DG size is given below:

1. Load flow for base case without DG is carried out.
2. Optimum size of DG (P_{dgi}) for each bus is computed
3. The system loss for every bus is determined by placing DG of optimum size (P_{dgi}) obtained in step 2 for that bus. The DG capacity is added to that bus as injection.
4. The bus for which lowest losses occur is selected as best location and the sizing of DG is taken as optimum capacity of DG as computed in step 2 for that bus.

VI. VOLTAGE STABILITY PERSPECTIVE VERSUS POWER LOSS REDUCTION PERSPECTIVE

It can be observed that the approach to maximise Voltage Stability does not account for power loss minimization. In the same way, the approach for loss minimization does not account for maximizing voltage stability. However for efficient operation of the network the optimum location and sizing of DG sources should be arrived at by taking all these parameters into account simultaneously. The computation of DG location and size by considering either voltage stability or power loss reduction independently will not lead to the best solution. In order to overcome this drawback the proposed approach with computation of NPEI accounts for overall system performance improvement while selection of DG location and size.

VII. SYSTEM PERFORMANCE INDICES

A. Voltage Stability Index

The decline of voltage stability level is one of important factors which restrict the increase of load served by distribution companies. DGs connected to distribution networks are potential to improve the system voltage stability. This voltage stability index is used to represent the index for voltage stability in NPEI and defined as the ratio of reduction in voltage stability index with insertion of DG at bus 'i' to the maximum or optimum reduction in VSI with insertion of DG in the system.

$$VSIn_i = \frac{\left(VSI_0 - VSI_{dgi}\right)}{\left(VSI_0 - VSI_{dg}\right)_{Opt}} \tag{7}$$

Where, VSI_0 is the voltage stability index of the distribution network without DG and VSI_{dgi} is the voltage stability index of the network with DG at node 'i'.

B. Power Loss Reduction Index

One of the major potential benefits offered by DG is the reduction in electrical line losses. The loss can be significant under heavy load conditions. The utility is forced to pass the cost of electrical line losses to all customers in terms of higher energy cost. With the inclusion of DG, line loss in the distribution system can be reduced. The proposed index for a bus is defined as the ratio of reduction in line losses in the system with DG insertion at bus 'i' to the maximum reduction in line losses with optimum size of DG in the system and is expressed as

$$PLossIn = \frac{\left(PLoss_0 - PLoss_{dgi}\right)}{\left(PLoss_0 - PLoss_{dg}\right)_{Opt}} \tag{8}$$

C. Voltage Profile Improvement Index

One of the justifications for introducing DG is to improve the voltage profile of the system and maintain the voltage at customer terminals within an acceptable range. Voltage profile can be improved because DG can provide a portion of the real and reactive power to the load, thus helping to decrease current along a section of the distribution line, which, in turn, will result in a boost in the voltage magnitude at the customer site. The voltage profile index for i^{th} node is defined as below [3]:

$$VP_i = \frac{\left(V_i - V_{\min}\right)\cdot\left(V_{\max} - V_i\right)}{\left(V_{nom} - V_{\min}\right)\cdot\left(V_{\max} - V_{nom}\right)} \tag{9}$$

Where VPi is the voltage profile of the i^{th} bus, and Vmin and Vmax are the minimum and maximum permissible voltages of the system nodes and Vnom is the nominal or desired bus voltage, typically taken as 1 pu .

The voltage profile index of the system is defined as :

$$VPIn = \frac{1}{n}\cdot\sum_{i=1}^{n} VP_i \tag{10}$$

Voltage profile Improvement Index (VPII) has been defined as the ratio of a measure of the voltage profile of the system with DG to the same measure with no DG employed and given as:

$$VPII_i = \frac{VPIn_{dgi}}{VPIn} \tag{11}$$

Where $VPIn_{dgi}$ is the voltage profile index of the system with DG at i^{th} bus and VPIn is the voltage profile index of the system without DG. As the value of this index is more than 1, it is modified appropriately for incorporation in the proposed Network Performance Enhancement Index as below:

$$VPIIn_i = \frac{VPII_i - 1}{VPII_{\max} - 1} \tag{12}$$

D. Reduction in Power Loss Per DG size Index

In the planning of Distributed Generator for a distribution network, the size of the DG plays a vital role. As the size of the DG is directly related to economics, it is advantageous for the utilities to get maximum reduction in power loss for lesser size of DG, that is reduction in Power Loss per unit size of DG which can be termed as (redPLoss/Pdg). Hence an index pertaining to this parameter is given as

$$redPLoss/PdgIn = \frac{\left(\frac{PLoss_0 - PLoss_{dgi}}{P_{dgi}}\right)}{\left(\frac{PLoss_0 - PLoss_{dg}}{P_{dg}}\right)_{\max}} \tag{13}$$

Where $PLoss_0$ is the Power Loss of the system without DG, $PLoss_{dgi}$ is the power loss of the system with DG at i^{th} node, P_{dgi} is the size of the Distributed Generator at i^{th} node.

E. Network Performance Enhancement Index(NPEI)

A composite index termed as Network Performance Enhancement Index is proposed to quantify the benefits of Distributed Generator like power loss reduction, voltage stability improvement, Voltage profile Improvement and maximum reduction of power loss per unit size of DG in a combined manner. The NPEI for any node is computed as

NPEI = W1 (VSIn) + W2 (PLossIn) + W3 (VPIIn)
+W4 (redPLoss/PdgIn)

Where W1, W2,W3 and W4 are the weithage factors such that 0≤ W1≤ 1, 0≤ W2≤ 1 , 0≤ W3≤ 1, 0≤ W4≤ 1 and

W1+W2+W3+W4 =1.

The Weightage factors are decided by the designer of the distribution system. The parameter which has been given highest weightage factor will get largest improvement after insertion of DG. In addition all other parameters included in NPEI also get improvement over the base case. Hence the Selection of the best bus for location of DG is made by highest value of NPEI. Such a selection will result in comprehensive improvement in network performance like reduction in system power loss, improvement in voltage stability, improvement in voltage profile. The system improvement is done with economical size of DG since NPEI includes factor for maximum reduction in system losses per unit size of DG. A priority list with decreasing value of NPEI is prepared so that DG can be inserted at the most feasible location. The top priority location gives highest value of NPEI, improves the system performance with lowest capacity of DG source. Hence this is the best and economical solution. As NPEI value decreases, the capacity of DG source to be inserted increases hence assigned the next lower priority levels. Incase the first priority with highest NPEI is not feasible then the designer can select the next best location with decreasing order of priority. This process gives flexibility and alternatives for efficient planning of the network.

VIII. Heuristic Approach For Allocation Of Dg

The flow chart for heuristic approach for allocation of DG source is given below:

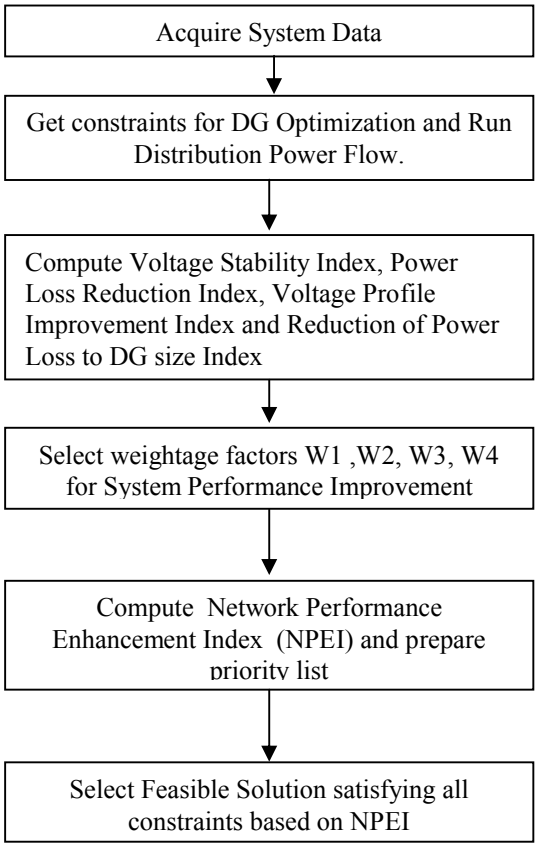

Figure 1. Flow for optimization of DG Allocation.

The designer needs to select the most feasible option for selection of DG location and its capacity. The constraints posed for decision could be from geographical, limited resources and economic domains. The geographical constraints include, right of way issues, availability of required land space, environmental issues etc. The availability of required type resources in the vicinity also plays a major role in decision making process. The economic constraint limits the capacity of the DG source to be connected to a magnitude lower than the optimal capacity of DG. The non-optimal capacity of DG will not maximize the benefit. The proposed approach incorporates these factors to facilitate the designer to choose the most feasible location and economical capacity to ensure improvement in the system performance. This approach overcomes the drawback of concentrating on only one parameter, for instance reduction in power loss in deciding the location of DG, while not giving attention to voltage stability factors or voltage profile improvement factor. Since the proposed approach includes comprehensively all the factors required for efficient operation of the distribution system, it ensures that the insertion of DG at the most beneficial location such that all the required parameters are better than the base system without DG. In addition the priority list obtained with decreasing value of NPEI will help the designer to select the most feasible location for insertion of DG to the system.

IX. Case Studies And Results

This approach is tested on two test systems 33 bus and 90 bus systems [1] [6]. For each system the optimum location and size of the distributed generation source is determined based on NPEI and the bus with highest NPEI is assigned with highest priority and as NPEI decreases priority also decreases. The designer can select most suitable location satisfying all constraints.

A. 33 Bus System

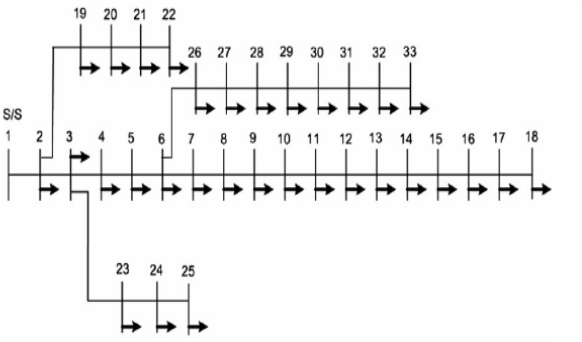

Figure 2. 33 Bus Test System.

The 33 bus system has total load of 3.72 MW and 2.3 MVAR. The base system has got real power loss of 211.20 kW and voltage stability index of 0.0748. When the power loss reduction perspective or voltage stability index perspective is individually considered, the parameter of interest is improved to larger extent compared to others. However with the help of NPEI all the network performance indicators get improvement than the base system as illustrated below:

1) Voltage Stability Improvement Perspective

By considering the perspective to improve the voltage stability to the largest extent that is to obtain the maximum

reduction in the value of Voltage Stability Index from its base value of 0.0748, the DG location and size is as follows:

TABLE I. DG CAPACITY ALLOCAITION WITH VOLTAGE STABILITY PERSPECTIVE

Location of DG (Bus No.)	Size of DG (MW)	Voltage Stability Index	System Losses (kW)	Voltage Profile Improvement Index
8	1.73	0.0445	83.51	1.191

As can be seen from the results, the voltage stability index has reduced to 0.0445 from its base case value of 0.0748 indicating improvement in voltage stability of the system. The Power loss also reduced to 83.51 kW, however this is not the highest reduction in the power loss possible as evident from the following analysis.

2) Power Loss Reduction Perspective

By considering the perspective to maximize reduction of the system power loss from its base case value of 211.20 kW, the DG location and size is as follows:

TABLE II. DG CAPACITY ALLOCAITION WITH POWER LOSS REDUCTION PERSPECTIVE

Location of DG (Bus No.)	Size of DG (MW)	Voltage Stability Index	System Losses (kW)	Voltage Profile Improvement Index
6	2.49	0.0512	68.04	1.188

The above result shows that insertion of DG at bus 6 with capacity of DG as 2.49 MW will lead to maximum reduction in losses bringing its value to 68.04 kW. However Voltage Stability Index is 0.0512 which is more than the earlier case value of 0.0442 indicating the voltage stability improvement is not to the highest extent as obtained with voltage stability improvement perspective. Also the higher capacity of DG as 2.49 MW needs to be inserted involving higher cost for DG.

3) Network Performance Enhancement Index Perspective

In order to give emphasis to all the above parameters responsible for network performance keeping the economical factor such as getting the maximum benefit with optimum size of the DG, the selection is done based on the proposed NPEI. As a sample case equal weightage is given to W1, W2, W3, and W4.

TABLE III. DG CAPACITY ALLOCAITION WITH NPEI PERSPECTIVE

Location of DG (Bus No.)	Size of DG (MW)	Voltage Stability Index	System Losses (kW)	Voltage Profile Improvement Index
30	1.47	0.0531	71.54	1.166

The bus number 30 is selected for insertion of DG since it has got highest NPEI computed as 0.8992. The results indicate that with a smaller capacity of DG of 1.47 MW all the parameters have improved than the base case. The voltage profile at all the buses is improved as illustrated in Fig. 3. Hence Selection of DG location and its size by NPEI is advantageous and economical than selection of DG from Voltage Stability perspective alone or power loss reduction perspective alone. The priority list of buses with decreasing value of NPEI is obtained from the developed algorithm. In this manner the proposed index NPEI will help the designer to select the most feasible location such that all the network performance parameters get improved with the economical size of DG.

TABLE IV. PRIORITY LIST BASED ON NPEI

Priority	NPEI	Location of DG (Bus No.)	Size of DG (MW)
1	0.8992	30	1.47
2	0.8504	8	1.73
3	0.8437	26	2.35

As evident from the priority list, the location with top priority is having highest NPEI and lowest capacity of DG to be inserted. Hence this is the economical as well as the best location from system performance improvement. As NPEI decreases, the size of DG increases, hence they are assigned later priority. If top priority location is not feasible for any reason then the next priority location can be selected.

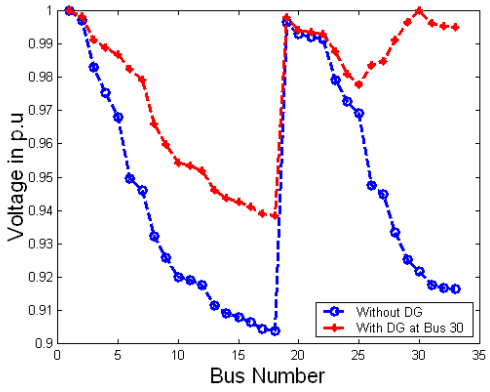

Figure 3. Voltage Profile Improvement with DG at Bus 30.

B. 90 Bus System

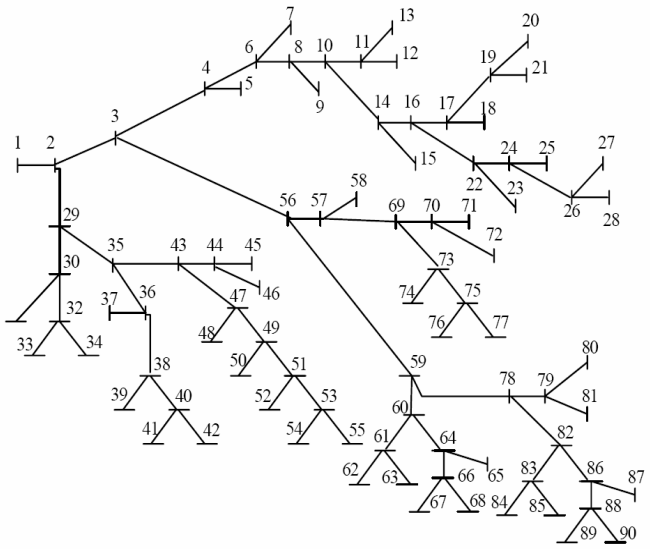

Figure 4. 90 Bus Test System.

The 90 bus system has total load of 19.45 MW and 29.72 MVAR. The base system has got real power loss of 0.504 MW and voltage stability index of 0.2367 The analysis of the 90 bus system is given in Table V to Table VIII below:

1) Voltage Stability Improvement Perspective

The results for obtaining the maximum reduction in the value of Voltage Stability Index from its base value of 0.2367, the DG location and size is as follows:

TABLE V. DG CAPACITY ALLOCAITION WITH VOLTAGE STABILITY PERSPECTIVE

Location of DG (Bus No.)	Size of DG (MW)	Voltage Stability Index	System Losses (MW)	Voltage Profile Improvement Index

42	2.13	0.1059	0.393	1.039

As can be seen from the results, the voltage stability index has reduced to 0.1059 from its base case value of 0.2367 indicating improvement of voltage stability of the system. The Power loss also reduced to 0.393 MW, however this is not the highest reduction in the power loss possible as evident from the following analysis.

2) Power Loss Reduction Perspective

The results to determine maximum reduction in the system power loss from its base case value of 0.504 MW, the DG location and size is as follows:

TABLE VI. DG CAPACITY ALLOCAITION WITH POWER LOSS REDUCTION PERSPECTIVE

Location of DG (Bus No.)	Size of DG (MW)	Voltage Stability Index	System Losses (MW)	Voltage Profile Improvement Index
10	10.98	0.2289	0.237	1.164

The above result shows that insertion of DG at bus 10 with capacity of DG as 10.98 MW will lead to maximum reduction in losses bringing its value to 0.237 MW. However Voltage Stability Index is 0.2289 which is more than the earlier case value of 0.1059 indicating the voltage stability improvement is not to the highest extent as obtained with voltage stability improvement perspective. Also the higher capacity of DG as 10.98 MW needs to be inserted involving higher cost for DG.

3) Network Performance Enhancement Index Perspective

The proposed NPEI approach is applied to 90 bus system with equal weightage factors as a sample case.

TABLE VII. DG CAPACITY ALLOCAITION WITH NPEI PERSPECTIVE

Location of DG (Bus No.)	Size of DG (MW)	Voltage Stability Index	System Losses (MW)	Voltage Profile Improvement Index
16	6.91	0.2206	0.256	1.155

The bus number 16 has got highest value of NPEI computed as 0.6615. The voltage profile at all the buses is improved as illustrated in Fig. 5.

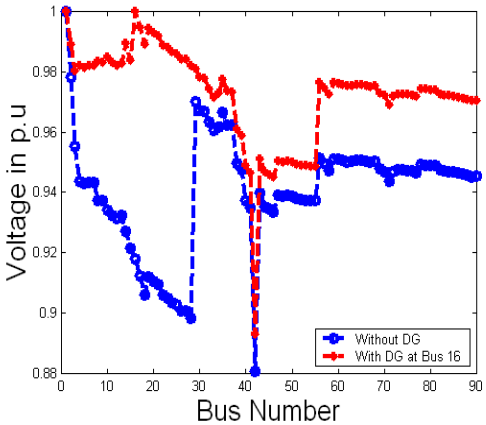

Figure 5. Voltage Profile Improvement with DG at Bus 16.

As indicated in the above result with insertion of much lower size of DG of 6.91 MW at bus number 16, all the network parameters have improved than the base case. Hence the proposed approach of selection of location and size of DG with network performance enhancement index will give comprehensive improvement to parameters like power loss reduction, voltage stability improvement, voltage profile improvement, with the lower size of DG than the other approaches of voltage stability perspective alone or power loss reduction perspective alone. In this regard with the help of NPEI, the designer is having the flexibility to select the DG location and size taking the economical constraints also to account.

TABLE VIII. PRIORITY LIST BASED ON NPEI

Priority	NPEI	Location of DG (Bus No.)	Size of DG (MW)
1	0.6615	16	6.91
2	0.6541	14	8.87
3	0.6348	8	11.35

It can be observed that, as NPEI decreases the size of DG to be inserted increases. The best location with top priority is having highest NPEI indicating improvement in all system parameters and economical. The designer can select other alternatives based on priority depending up on the scenario and requirement.

X. CONCLUSION

The distribution sector reforms have made significant impact for operation and management of utility system. The distributed generation sources are playing a key role in meeting the load demand, reducing the power losses and overall improvement in network management. This paper proposes a heuristic approach for determination of DG allocation incorporating the constraints to arrive at a feasible solution. The Network Performance Enhancement Index (NPEI) serves as an indicator for best possible choice. The designer will be able to select the optimum solution satisfying all the constraints with the help of NPEI. A priority list based on NPEI is formed to select the most feasible and economical solution. The developed approach is tested on 33 bus and 90 bus systems and results obtained have shown that the technique gives the better and economical solution for system improvement.

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