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BALL CONVERGENCE OF NEWTON'S METHOD FOR GENERALIZED EQUATIONS USING RESTRICTED CONVERGENCE DOMAINS AND MAJORANT CONDITIONS

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Abstract. In this study, we consider Newton's method for solving the generalized equation of the form $F(x) + T(x) \ni 0$, in Hilbert space, where F is a Fréchet differentiable operator and T is a set valued and maximal monotone. Using restricted convergence domains and Banach Perturbation lemma we prove the convergence of the method with the following advantages: tighter error estimates on the distances involved and the information on the location of the solution is at least as precise. These advantages were obtained under the same computational cost but using more precise majorant functions.

1. Introduction

In this study we consider the problem of approximately solving the generalized equation

$$F(x) + Q(x) \ni 0, \tag{1.1}$$

where $F : D \longrightarrow H$ is a nonlinear Fréchet differentiable operator defined on the open subset D of the Hilbert space H and $Q : H \rightrightarrows H$ is set-valued and maximal monotone. It is well known that many problems of practical interest

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can be modeled into an equation of the form (1.1) [17]-[22], [27, 28, 30, 36]. If $\psi : H \longrightarrow (-\infty, +\infty]$ is a strict lower semi-continuous convex function and

$$Q(x) = \partial\psi(x) = \{u \in H : \psi(y) \geq \psi(x) + \langle u, y - x\rangle\}, \ \ \forall\, y \in H,$$

then (1.1) becomes the variational inequality problem

$$F(x) + \partial\psi(x) \ni 0,$$

including linear and nonlinear complementary problems, additional comments about such problems can be found in [1]-[37].

In the present paper, we consider Newton's method defined for each $n = 0, 1, 2, \cdots$ by

$$F(x_k) + F'(x_k)(x_{k+1} - x_k) + F(x_{k+1}) \ni 0 \tag{1.2}$$

for approximately solving (1.1). We will use the idea of restricted convergence domains to present a convergence analysis of (1.2). In our analysis we relax the Lipschitz type continuity of the derivative of the operator involved. The basic idea of the analysis is to find larger convergence domain for the method (1.2). Using the restricted convergence domains, we obtained a finer convergence analysis, with the advantages (**A**): tighter error estimates on the distances involved and the information on the location of the solution is at least as precise. These advantages were obtained (under the same computational cost) using the same or weaker hypotheses as in [33].

The rest of the paper is organized as follows. Section 2 contains the necessary background needed. In section 3, we present the local convergence. The numerical examples are presented in the concluding section 4.

2. Preliminaries

In order to make the study as self contained as possible we reintroduce some standard notations and auxiliary results for the monotonicity of set valued operators [18, 22, 27]. Denote by $U(w, \xi)$, $\overline{U}(w, \xi)$, the open and closed balls in H, respectively, with center $w \in H$ and of radius $\xi > 0$.

Next, we recall notions of monotonicity for set-valued operators.

Definition 2.1. Let $Q : H \rightrightarrows H$ be a set-valued operator. Q is said to be monotone if for any $x, y \in domQ$ and $u \in Q(y), v \in Q(x)$ implies that the following inequality holds:

$$\langle u - v, y - x\rangle \geq 0.$$

A subset of $H \times H$ is monotone if it is the graph of a monotone operator. If $\varphi : H \longrightarrow (-\infty, +\infty]$ is a proper function then the subgradient of φ is monotone.

Definition 2.2. Let $Q : H \rightrightarrows H$ be monotone. Then Q is maximal monotone if the following holds for all $x, u \in H$:

$$\begin{aligned} &\langle u - v, y - x\rangle \geq 0 \quad \text{for each} \quad y \in domQ \text{ and } v \in Q(y) \\ &\Longrightarrow x \in domQ \quad \text{and} \quad v \in Q(x). \end{aligned}$$

We will be using the following results for proving our results.

Lemma 2.3. *Let G be a positive operator (i.e. $\langle G(x), x\rangle \geq 0$). The following statements about G hold:*

- $\|G^2\| = \|G\|^2$.
- *If G^{-1} exists, then G^{-1} is a positive operator.*

Lemma 2.4. *Let G be a positive operator. Suppose that G^{-1} exists, then for each $x \in H$ we have*

$$\langle G(x), x\rangle \geq \frac{\|x\|^2}{\|G^{-1}\|}.$$

Lemma 2.5. *Let $B : H \longrightarrow H$ be a bounded linear operator and $I : H \longrightarrow H$ the identity operator. If $\|B - I\| < 1$ then B is invertible and $\|B^{-1}\| \leq \frac{1}{(1-\|B-I\|)}$.*

Let $G : H \longrightarrow H$ be a bounded linear operator. Then $\widehat{G} := \frac{1}{2}(G+G^*)$ where G^* is the adjoint of G. Hereafter, we assume that $Q : H \rightrightarrows H$ is a set valued maximal monotone operator and $F : H \longrightarrow H$ is a Fréchet differentiable operator.

3. Local convergence

In this section, we study the local convergence of the Newton's method for solving the generalized equation (1.1) based on the partial linearization of (1.1) ([30]). Our main result is the following:

Theorem 3.1. *Let $F : D \subset H \longrightarrow H$ be nonlinear operator with a continuous Fréchet derivative F', where D is an open subset of H. Let $Q : H \rightrightarrows H$ be a set-valued operator and $x^* \in D$. Suppose that $0 \in F(x^*) + Q(x^*), F'(x^*)$ is a positive operator and $\widehat{F'(x^*)}^{-1}$ exists. Let $R > 0$ and suppose that there exist $f_0, f : [0, R) \longrightarrow \mathbb{R}$ twice continuously differentiable such that*

(h_0) *for $x \in D$,*

$$\|\widehat{F'(x^*)}^{-1}\|\|F'(x) - F'(x^*)\| \leq f_0'(\|x - x^*\|) - f_0'(0), \tag{3.1}$$

and

$$\|\widehat{F'(x^*)}^{-1}\|\|F'(x) - F'(x^* + \theta(x - x^*))\| \le f'(\|x - x^*\|) - f'(\theta\|x - x^*\|) \tag{3.2}$$

for each $x \in D_0 = D \cap U(x^*, R)$, $\theta \in [0, 1]$.

(h_1) $f(0) = f_0(0)$ *and* $f'(0) = f_0'(0) = -1$, $f_0(t) \le f(t)$, $f_0'(t) \le f'(t)$ *for each* $t \in [0, R)$.

(h_2) f_0', f' *are convex and strictly increasing.*

Let

$$\nu := \sup\{t \in [0, R) : f'(t) < 0\}$$

and

$$r := \sup\left\{t \in (0, \nu) : \frac{f(t)}{tf'(t)} - 1 < 1\right\}.$$

Then the sequence with starting point $x_0 \in B(x^*, r)/\{x^*\}$ *and* $t_0 = \|x^* - x_0\|$, *respectively,*

$$0 \in F(x_k) + F'(x_k)(x_{k+1} - x_k) + Q(x_{k+1}),$$
$$t_{k+1} = |t_k - \frac{f(t_k)}{f'(t_k)}|, \quad k = 0, 1, \cdots, \tag{3.3}$$

are well defined, $\{t_k\}$ *is strictly decreasing, is contained in* $(0, r)$ *and converges to* 0, $\{x_k\}$ *is contained in* $U(x^*, r)$ *and converges to the point* x^* *which is the unique solution of the generalized equation* $F(x) + Q(x) \ni 0$ *in* $U(x^*, \bar{\sigma})$, *where* $\bar{\sigma} = \min\{r, \sigma\}$ *and* $\sigma := \sup\{0 < t < R : f(t) < 0\}$. *Moreover, the sequence* $\{\frac{t_{k+1}}{t_k^2}\}$ *is strictly decreasing,*

$$\|x^* - x_{k+1}\| \le \left[\frac{t_{k+1}}{t_k^2}\right]\|x_k - x^*\|^2, \quad \frac{t_{k+1}}{t_k^2} \le \frac{f''(t_0)}{2|f'(t_0)|}, \quad k = 0, 1, \cdots. \tag{3.4}$$

If, additionally $\frac{\rho f'(\rho) - f(\rho)}{\rho f'(\rho)} = 1$ *and* $\rho < R$, *then* $r = \rho$ *is the optimal convergence radius. Furthermore, for* $t \in (0, r)$ *and* $x \in \bar{U}(x^*, t)$,

$$\begin{aligned} \|x_{k+1} - x^*\| &\le \frac{e_f(\|x_k - x^*\|, 0)}{|f_0'(\|x_k^* - x^*\|)|} \\ &:= \alpha_k \\ &\le \frac{e_f(\|x_k - x^*\|, 0)}{|f'(\|x_k^* - x^*\|)|} \le \frac{|\eta_f(t)|}{t^2}\|x_k - x^*\|^2, \end{aligned} \tag{3.5}$$

where

$$e_f(s, t) := f(t) - (f(s) + f'(s)(t - s)) \quad \text{for each} \quad s, t \in [0, R)$$

and

$$\eta_f(t) := t - \frac{f(t)}{f'(t)} \quad \text{for each} \quad s, t \in [0, \nu).$$

Finally, by the second inequality in (3.5) *there exists* $r^* \geq r$ *such that*

$$\lim_{k \longrightarrow \infty} x_k = x^*,$$

if $x_0 \in U(x^*, r^*) - \{x^*\}$.

From now on we assume that the hypotheses of Theorem 3.1 hold.

Remark 3.2. The introduction of the center-Lipschitz-type condition (3.1) (*i.e.*, function f_0) leads to the introduction of restricted Lipschitz-type condition (3.2). The condition used in earlier studies [33] is given by

$$\|\widehat{F'(x^*)}^{-1}\| \|F'(x) - F'(x^* + \theta(x - x^*))\| \leq f_1'(\|x - x^*\|) - f_1'(\theta\|x - x^*\|) \quad (3.6)$$

for each $x \in D$, $\theta \in [0,1]$, where $f_1 : [0, +\infty) \longrightarrow \mathbb{R}$ is also twice continuously differentiable. It follows from (3.1), (3.2) and (3.6) that

$$f_0'(t) \leq f_1'(t), \quad (3.7)$$

$$f(t) \leq f'(t) \quad (3.8)$$

for each $t \in [o, \nu)$, since $D_0 \subseteq D$. If $f_0'(t) = f_1'(t) = f'(t)$ for each $t \in [0, \nu)$, then our results reduce to the corresponding ones in [33]. Otherwise, (*i.e.*, if strict inequality holds in (3.7) or (3.8)) then the new results improve the old ones. Indeed, let

$$r_1 := \sup\left\{t \in (0, \bar{\nu}) : -\frac{tf_1'(t) - f_1(t)}{tf_1'(t)} < 1\right\},$$

where $\nu_1 := \sup\{t \in [0, +\infty) : f_1'(t) < 0\}$. Then, the error bounds are (corresponding to (3.5)):

$$\begin{aligned} \|x_{k+1} - x^*\| &\leq \frac{e_{f_1}(\|x_k - x^*\|, 0)}{|f_1'(\|x_k^* - x^*\|)|} \\ &:= \beta_k \leq \frac{|\eta_{f_1}(t)|}{t^2}\|x_k - x^*\|^2. \end{aligned} \quad (3.9)$$

In view of the definition of r, r_1 and estimates (3.5), (3.7), (3.8) and (3.9), we deduce that

$$r_1 \leq r \quad (3.10)$$

and

$$\alpha_k \leq \beta_k, \ k = 1, 2, \cdots. \quad (3.11)$$

Hence, we obtain a larger radius of convergence and tighter error estimates on the distances involved, leading to a wider choice of initial guesses x^* and fewer computations of iterates x_k in order to achieve a desired error tolerance. It is also worth noticing:

The advantages are obtained under the same computational cost, since in practice the computation of function f_1 requires the computation of function

f_0 and f as special cases. The introduction of function f was not possible before (when only function f_1 was used). This introduction become possible using function f_0 (i.e., f is a function of f_0) (see also the numerical examples).

Next, we present an auxiliary Banach Lemma relating the operator F with the majorant function f_0.

Lemma 3.3. *Assume: there exists* $x^* \in H$ *such that* $\widehat{F'(x^*)}$ *is a positive operator and* $\widehat{F'(x^*)}^{-1}$ *exists;* $\|x - x^*\| \le \min\{R, \nu\}$. *Then,* $\widehat{F'(x^*)}$ *is a positive operator* $\widehat{F'(x^*)}^{-1}$ *exists and*

$$\|\widehat{F'(x)}^{-1}\| \le \frac{\|\widehat{F'(x^*)}^{-1}\|}{|f_0'(\|x - x^*\|)|}. \tag{3.12}$$

Proof. The proof follows as in [33] but there are vital differences, when the needed function f_0 is used instead of the less precise f. Notice that

$$\begin{aligned}\|\widehat{F'(x)} - \widehat{F'(x^*)}\| &\le \frac{1}{2}\|F'(x) - F'(x^*)\| + \frac{1}{2}\|(F'(x) - F'(x^*))^*\| \\ &= \|F'(x) - F'(x^*)\|.\end{aligned} \tag{3.13}$$

Let $x \in U(x^*, r)$. If follows that $\|x - x^*\| < \nu$, since $r < \nu$. Consequently, $f_0'(\|x - x^*\|) < 0$. Using (h_0) and (3.13) we get that

$$\begin{aligned}\|\widehat{F'(x^*)}^{-1}\|\|\widehat{F'(x)} - \widehat{F'(x^*)}\| &\le \|\widehat{F'(x^*)}^{-1}\|\|F'(x) - F'(x^*)\| \\ &\le f_0'(\|x - x^*\|) - f_0'(0) < 1\end{aligned} \tag{3.14}$$

for all $x \in U(x^*, r)$. It follows from (3.14) and the Banach Lemma, that $\widehat{F'(x^*)}^{-1}$ exists. Moreover, in view of (3.14)

$$\begin{aligned}\|\widehat{F'(x)}^{-1}\| &\le \frac{\|\widehat{F'(x^*)}^{-1}\|}{1 - \|\widehat{F'(x^*)}^{-1}\|\|F'(x) - F'(x^*)\|} \\ &\le \frac{\|\widehat{F'(x^*)}^{-1}\|}{1 - (f_0'(\|x - x^*\|) - f_0(0))} \\ &= \frac{\|\widehat{F'(x^*)}^{-1}\|}{|f_0'(\|x - x^*\|)|},\end{aligned}$$

since $r = \min\{R, \nu\}$. Furthermore, using (3.14) we have

$$\|\widehat{F'(x)} - \widehat{F'(x^*)}\| \le \frac{1}{\|\widehat{F'(x^*)}^{-1}\|}. \tag{3.15}$$

We also from the above inequality for $y \in H$ that

$$\langle (\widehat{F'(x^*)} - \widehat{F'(x)})y, y\rangle \le \|\widehat{F'(x^*)} - \widehat{F'(x)}\| \|y\|^2 \le \frac{\|y\|^2}{\|\widehat{F'(x^*)}^{-1}\|},$$

leading to

$$\widehat{F'(x^*)}y, y\rangle - \frac{\|y\|^2}{\|\widehat{F'(x^*)}^{-1}\|} \le \langle \widehat{F'(x)}y, y\rangle.$$

In view of Lemma 2.5, we get that

$$\langle \widehat{F'(x^*)}y, y\rangle \ge \frac{\|y\|^2}{\|\widehat{F'(x^*)}^{-1}\|}.$$

Hence, by the two last inequalities we deduce that $\langle \widehat{F'(x)}y, y\rangle \ge 0$, *i.e.* $\widehat{F'(x)}$ is a positive operator. Lemma 2.4 shows that $\widehat{F'(x)}$ is a positive operator and $\widehat{F'(x)}^{-1}$ exists. Hence, by Lemma 2.3 we have that

$$\langle \widehat{F'(x)}y, y\rangle \ge \frac{\|y\|^2}{\|\widehat{F'(x)}^{-1}\|}$$

for any $y \in H$. □

We can write that $\langle \widehat{F'(x)}y, y\rangle = \langle F'(x)y, y\rangle$. Then, by the second part of Lemma 2.4 we conclude that the Newton iteration mapping is well-defined. Denote by N_{F+Q}, the Newton iteration mapping for $f + F$ in that region. In particular, $N_{F+Q} : U(x^*, r) \longrightarrow H$ is defined by

$$0 \in F(x) + F'(x)(N_{F+Q}(x) - x) + Q(N_{F+Q}(x)),\ \forall\, x \in U(x^*, r). \tag{3.16}$$

Remark 3.4. Under condition (3.6) it was shown in [33] that

$$\|\widehat{F'(x)}^{-1}\| \le \frac{\|\widehat{F'(x^*)}^{-1}\|}{|f_1'(\|x - x^*\|)|} \tag{3.17}$$

instead of (3.12). However, we have that (3.12) gives a tighter error estimate than (3.13), since $|f_1'(t) \le |f_0'(t)|$. This is a crucial difference in the proof of Theorem 3.1.

Proof of Theorem 3.1. Simply follow the proof of Theorem 4 in [33] but notice that the iterates x_k lie in D_0 which is a more precise location than D (used in [33]) allowing the usage of tighter function f than f_1 and also the usage of tighter function f_0 that f_1 for the computation of the upper bounds of the inverses $\|\widehat{F'(x)}^{-1}\|$ (*i.e.*, we use (3.12) instead of (3.17)). □

4. Special Cases

Although in Remark 3.4, we have shown the advantages of our new approach over earlier ones, we also compare our results in the special case of the Kantorovich theory [8, 11, 27, 29] with the corresponding ones in [16, 29], Rall [16, 29], Traub and Wozniakowski [35], when $F \equiv \{0\}$. Similar favorable comparisons can be given in the special case of Smale's theory [34] or Wang's theory [37]. Let functions f_0, f, f_1 be defined by

$$f_0'(t) = \frac{L_0}{2}t^2 - t,$$

$$f'(t) = \frac{L}{2}t^2 - t$$

and

$$f_1'(t) = \frac{L_1}{2}t^2 - t$$

for some positive constants L_0, L and L_1 to be determined using a specialized operator F.

Example 4.1. Let $X = Y = \mathbb{R}^3$, $D = \bar{U}(0,1)$, $x^* = (0,0,0)^T$. Define function F on D for $w = (x,y,z)^T$ by

$$F(w) = \left(e^x - 1, \frac{e-1}{2}y^2 + y, z\right)^T.$$

Then the Fréchet-derivative is given by

$$F'(v) = \begin{bmatrix} e^x & 0 & 0 \\ 0 & (e-1)y+1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Notice that $L_0 = e - 1$, $L = e^{\frac{1}{L_0}}$, $L_1 = e$ and hence $f_0(t) < f(t) < f_1(t)$. Therefore, we have that the conditions of Theorem 3.1 hold. Moreover, we have that

$$r_1 := \frac{2}{3L_1} < r := \frac{2}{3L} < r^* := \frac{2}{2L_0 + L}.$$

Furthermore, the corresponding error bounds are:

$$\|x_{k+1} - x^*\| \leq \frac{L\|x_k - x^*\|^2}{2(1 - L_0\|x_k - x^*\|)},$$

$$\|x_{k+1} - x^*\| \leq \frac{L\|x_k - x^*\|^2}{2(1 - L\|x_k - x^*\|)}$$

and

$$\|x_{k+1} - x^*\| \leq \frac{L_1\|x_k - x^*\|^2}{2(1 - L_1\|x_k - x^*\|)}.$$

References

[1] I.K. Argyros, *A semilocal convergence analysis for directional Newton methods*, Mathematics of Computation, AMS, **80** (2011), 327–343.

[2] I.K. Argyros, *Computational theory of iterative methods*, Studies in Computational Mathematics, **15**, Editors: K. Chui and L. Wuytack. Elsevier (2007) New York, U.S.A.

[3] I.K. Argyros, *Concerning the convergence of Newton's method and quadratic majorants*, J. Appl. Math. Comput., **29** (2009), 391–400.

[4] I.K. Argyros and D. González, *Local convergence analysis of inexact Gauss-Newton method for singular systems of equations under majorant and center-majorant condition*, SeMA, **69**(1) (2015), 37–51.

[5] I.K. Argyros and S. Hilout, *On the Gauss-Newton method*, J. Appl. Math., (2010), 1–14.

[6] I.K. Argyros and S. Hilout, *Extending the applicability of the Gauss-Newton method under average Lipschitz-conditions*, Numer. Alg., **58** (2011), 23–52.

[7] I.K. Argyros and S. Hilout, *On the solution of systems of equations with constant rank derivatives*, Numer. Algor., **57** (2011), 235–253.

[8] I.K. Argyros, Y.J. Cho and S. Hilout, *Numerical methods for equations and its applications*, CRC Press/Taylor and Francis Group, New-York (2012).

[9] I.K. Argyros and S. Hilout, *Improved local convergence of Newton's method under weaker majorant condition*, J. Comput. Appl. Math., **236**(7) (2012), 1892–1902.

[10] I.K. Argyros and S. Hilout, *Weaker conditions for the convergence of Newton's method*, J. Complexity, **28** (2012), 364–387.

[11] I.K. Argyros and S. Hilout, *Computational Methods in Nonlinear Analysis*, World Scientific Publ. Comp., New Jersey (2013).

[12] L. Blum, F. Cucker, M. Shub and S. Smale. *Complexity and real computation*, Springer-Verlag, New York, 1998. With a foreword by Richard M. Karp.

[13] L. Blum, F. Cucker, M. Shub and S. Smale, *Complexity and Real Computation*, Springer-Verlag, New York (1997).

[14] D.C. Chang, J. Wang and J.C. Yao, *Newtons method for variational inequality problems: Smales point estimate theory under the γ-condition*, Applicable Analysis, **94**(1) (2015), 4455.

[15] J.E. Dennis Jr. and R.B. Schnabel, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, in: Classics in Applied Mathematics, SIAM, Philadelphia (1996).

[16] P. Deuflhard and G. Heindl, *Affine invariant convergence for Newtons method and extensions to related methods*, SIAM J. Numer. Anal., **16**(1) (1979), 1-10.

[17] A.L. Dontchev and R.T. Rockafellar, *Implicit Functions and Solution Mappings. A View From Variational Analysis*, in: Springer Monographs in Mathematics, Springer, Dordrecht (2009).

[18] S.P. Dokov and A.L. Dontchev, *Robinsons strong regularity implies robust local convergence of Newtons method Optimal control* (Gainesville, FL, 1997), vol. 15 of Appl. Optim., pages 116129 Kluwer Acad. Publ., Dordrecht (1998).

[19] A.L. Dontchev, *Local analysis of a Newton-type method based on partial linearization*, In The mathematics of numerical analysis (Park City, UT, 1995). Amer. Math. Soc., Providence, RI, 1996.

[20] A.L. Dontchev, *Local convergence of the Newton method for generalized equations*, C. R. Acad. Sci. Paris S er. I Math., **322**(4) (1996), 327-331.

[21] A.L. Dontchev and R.T. Rockafellar, *Implicit functions and solution mappings.* Springer Monographs in Mathematics. Springer, Dordrecht, 2009. A view from variational analysis.

[22] A.L. Dontchev and R. T. Rockafellar, *Newtons method for generalized equations: a sequential implicit function theorem*, Math. Program., **123**(1, Ser.B) (2010), 139-159.

[23] O. Ferreira, *Local convergence of Newtons method in Banach space from the viewpoint of the majorant principle*, IMA J. Numer. Anal., **29**(3) (2009), 746-759.

[24] O.P. Ferreira, M.L.N. Goncalves and P.R. Oliveira, *Local convergence analysis of the Gauss-Newton method under a majorant condition*, J. Complexity., **27**(1) (2011), 111-125.

[25] O.P. Ferreira and B.F. Svaiter, *Kantorovichs majorants principle for Newtons method*, Comput. Optim. Appl., **42**(2) (2009), 213-229.

[26] O.P. Ferreira, *A robust semi-local convergence analysis of Newtons method for cone inclusion problems in Banach spaces under affine invariant majorant condition*, Journal of Computational and Applied Mathematics, **279** (2015), 318-335.

[27] M. Josephy, *Newtons Method for Generalized Equations and the PIES Energy Model*, University of WisconsinMadison (1979).

[28] A. Pietrus and C. Jean-Alexis, *Newton-secant method for functions with values in a cone*, Serdica Math. J., **39**(3-4) (2013), 271-286.

[29] L.B. Rall, *A Note on the Convergence of Newtons Method*, SIAM Journal on Numerical Analysis, **11**(1) (1974), 34-36.

[30] S.M. Robinson, *Extension of Newtons method to nonlinear functions with values in a cone*, Numer. Math., **19** (1972), 341-347.

[31] S.M. Robinson, *Strongly regular generalized equations*, Math. Oper. Res., **5**(1) (1980), 43-62.

[32] S.M. Robinson, *Normed convex processes*, Trans. Amer. Math. Soc., **174** (1972), 127-140.

[33] G.N. Silva, *Convergence of the Newton's method for generalized equations under the majorant condition*, arXiv:1603.05280v1[math.NA]16 March, 2016.

[34] S. Smale, *Newton method estimates from data at one point*, in: R. Ewing, K. Gross, C. Martin (Eds.), The Merging of Disciplines: New Directions in Pure, Applied and Computational Mathematics, Springer-Verlag, New York, 1986, pp. 185196.

[35] J.F. Traub and H. Wozniakowski, *Convergence and complexity of Newton iteration for operator equations*, J. Assoc. Comput. Mach., **26**(2) (1979), 250-258.

[36] L.U. Uko, *Generalized equations and the generalized Newton method*, Math. Programming, **73**(3, Ser. A) (1996), 251-268.

[37] X. Wang, *Convergence of Newtons method and uniqueness of the solution of equation in Banach space*, IMA J. Numer. Anal., **20** (2000), 123-134.